

Name: Solution I.D. # _____ Section # _____1. Find the general solution of the system $X' = AX$, where $A = \begin{bmatrix} -1 & -1 & 0 \\ -2 & 2 & -1 \\ 0 & -2 & -1 \end{bmatrix}$

$$|A - \lambda I| = 0$$

$$\Rightarrow \begin{vmatrix} -1-\lambda & -1 & 0 \\ -2 & 2-\lambda & -1 \\ 0 & -2 & -1-\lambda \end{vmatrix} = 0$$

$$(-1-\lambda)[(2-\lambda)(-1-\lambda)-2] + [-2(-1-\lambda)] = 0$$

$$= (\lambda+1)(\lambda-3)(\lambda+2) = 0 \Rightarrow \lambda = -1, -2, 3$$

$$\underline{\underline{\lambda = -1}} \quad (A - \lambda I)K = 0$$

$$\Rightarrow \left[\begin{array}{ccc|c} 0 & -1 & 0 & 0 \\ -2 & 3 & -1 & 0 \\ 0 & -2 & 0 & 0 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} -2 & 3 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & -2 & 0 & 0 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} 1 & -\frac{3}{2} & \frac{1}{2} & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right] \Rightarrow k_2 = 0, k_1 = -\frac{1}{2}k_3$$

$$\therefore K_1 = \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}, \quad X_1 = \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix} e^{-t}$$

$$\underline{\underline{\lambda = -2}} \quad (A - \lambda I)K = 0$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ -2 & 4 & -1 & 0 \\ 0 & -2 & 1 & 0 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & 2 & -1 & 0 \\ 0 & -2 & 1 & 0 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & 1 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \end{array} \right] \Rightarrow k_2 = \frac{1}{2}k_3, k_1 = k_2$$

$$\therefore K_2 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \quad X_2 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} e^{-2t}$$

$$\underline{\underline{\lambda = 3}} \quad (A - \lambda I)K = 0$$

$$\left[\begin{array}{ccc|c} -4 & -1 & 0 & 0 \\ -2 & -1 & -1 & 0 \\ 0 & -2 & -4 & 0 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} 1 & \frac{1}{4} & 0 & 0 \\ -2 & -1 & -1 & 0 \\ 0 & -2 & -4 & 0 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} 1 & \frac{1}{4} & 0 & 0 \\ 0 & -\frac{1}{2} & -1 & 0 \\ 0 & -2 & -4 & 0 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} 1 & \frac{1}{4} & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -2 & -4 & 0 \end{array} \right] \Rightarrow \left. \begin{array}{l} k_2 = -2k_3 \\ k_1 = -\frac{1}{4}k_3 \end{array} \right\}$$

$$\therefore K_3 = \begin{bmatrix} \frac{1}{4} \\ -2 \\ 1 \end{bmatrix}, \quad X_3 = \begin{bmatrix} \frac{1}{4} \\ -2 \\ 1 \end{bmatrix} e^{3t}$$

The general solution is $X = C_1 \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix} e^{-t} + C_2 \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} e^{-2t} + C_3 \begin{bmatrix} \frac{1}{4} \\ -2 \\ 1 \end{bmatrix} e^{3t}$