Math 260 Quiz # 7

Name: Solution Serial # ______

1. Use the method of undetermined coefficients to find the general solution of:

$y'' - 6y' + 9y = 6x^2 + 2 - 12e^{3x}$

First, we solve the associated hom. eq^n $y'' - 6y' + 9y = 0$

3 $\therefore \boxed{y_c = c_1 e^{3x} + c_2 x e^{3x}}$

$\lambda^2 - 6\lambda + 9 = 0 \Rightarrow (\lambda - 3)^2 = 0 \Rightarrow \lambda = 3, 3$

To find y_p, we put, as the first trial, $y_p = Ax^2 + Bx + C + De^{3x}$

But the last term is duplicated in y_c. Even if we multiply it by x we get xe^{3x} which is again duplicated in y_c. So, we multiply the last term by x^2 to get the operative form of a particular solution as

2 $\boxed{y_p = Ax^2 + Bx + C + Dx^2 e^{3x}} \Rightarrow y_p' = 2Ax + B + 3Dx^2 e^{3x} + 2Dxe^{3x}$

$y_p'' = 2A + 9Dx^2 e^{3x} + 12Dxe^{3x} + 2De^{3x}$

Substitute in the given DE: $y_p'' - 6y_p' + 9y_p = 6x^2 + 2 - 12e^{3x}$

2 $\Rightarrow 2A + 9Dx^2 e^{3x} + 12Dxe^{3x} + 2De^{3x} - 12Ax - 6B - 18Dx^2 e^{3x} - 12Dxe^{3x} +$

$+ 9Ax^2 + 9Bx + 9C + 9Dx^2 e^{3x} = 6x^2 + 2 - 12e^{3x}$

$\Rightarrow 9Ax^2 + (9B - 12A)x + (2A - 6B + 9C) + 2De^{3x} = 6x^2 + 2 - 12e^{3x}$

1 Equating Coefficients, we get: $9A = 6 \Rightarrow \boxed{A = \frac{2}{3}}$, $9B - 12A = 0 \Rightarrow \boxed{B = \frac{8}{9}}$,

$2A - 6B + 9C = 2 \Rightarrow 2(\frac{2}{3}) - 6(\frac{8}{9}) + 9C = 2 \Rightarrow \boxed{C = \frac{2}{3}}$, $2D = -12 \Rightarrow \boxed{D = -6}$.

Substitute for the constants in y_p above, we get

1 $\boxed{y_p = \frac{2}{3}x^2 + \frac{8}{9}x + \frac{2}{3} - 6x^2 e^{3x}}$

The general solution of the given DE is $y = y_c + y_p$

1 i.e. $y = c_1 e^{3x} + c_2 x e^{3x} + \frac{2}{3}x^2 + \frac{8}{9}x + \frac{2}{3} - 6e^{3x}$.