Math 260 Quiz # 4

Name: Solution Serial # ______

1. Determine whether the following set is a subspace of $R^2$:
$W = \{(x, y) \in R^2 : x^2 + y^2 = 1\}$

Let $v = (x, y) \in W$, and let $c \in \mathbb{R}$. So $x^2 + y^2 = 1$.

Then, $cv = c(x, y) = (cx, cy)$

and $(cx)^2 + (cy)^2 = c^2(x^2 + y^2) = c^2 \neq 1$ if $c \neq 0$. $\Rightarrow cv \notin W$.

For example, take $c = 2$ then $(cx)^2 + (cy)^2 = 4 \neq 1$. $\Rightarrow W$ is not a subspace.

2. Find the solution for the following system and then write the solution as a linear combination of two vectors $u$ and $v$ so that the solution space can be described as the set of all linear combinations of the form $su + tv$:

$$x_1 - 4x_2 + x_3 - 4x_4 = 0$$
$$x_1 + 2x_2 + x_3 + 8x_4 = 0$$
$$x_1 + x_2 + x_3 + 6x_4 = 0$$

$$\left[\begin{array}{cccc|c} 1 & -4 & 1 & -4 & 0 \\ 1 & 2 & 1 & 8 & 0 \\ 1 & 1 & 1 & 6 & 0 \end{array}\right] \xrightarrow[-R_1+R_3]{-R_1+R_2} \left[\begin{array}{cccc|c} 1 & -4 & 1 & -4 & 0 \\ 0 & 6 & 0 & 12 & 0 \\ 0 & 5 & 0 & 10 & 0 \end{array}\right]$$

$$\xrightarrow{-R_3+R_2} \left[\begin{array}{cccc|c} 1 & -4 & 1 & -4 & 0 \\ 0 & 1 & 0 & 2 & 0 \\ 0 & 5 & 0 & 10 & 0 \end{array}\right] \xrightarrow[-5R_2+R_3]{4R_2+R_1} \left[\begin{array}{cccc|c} 1 & 0 & 1 & 4 & 0 \\ 0 & 1 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$x_3, x_4$ are free variables. $x_2 = -2x_4$
$x_1 = -4x_4 - x_3$

$$\left.\begin{array}{l} x_3 = s \\ x_4 = t \end{array}\right\} \Rightarrow \begin{array}{l} x_2 = -2t \\ x_1 = -4t - s \end{array}$$

The solution is $\bar{X} = (x_1, x_2, x_3, x_4) = (-4t - s, -2t, s, t)$

$= (-4t, -2t, 0, t) + (-s, 0, s, 0)$

$= t(-4, -2, 0, 1) + s(-1, 0, 1, 0)$

$= tv + su$, where $v = (-4, -2, 0, 1)$
$u = (-1, 0, 1, 0)$