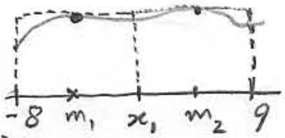
MATH 102 [Homework 1: **Due on** Wed., Feb. 23]

1. [**Use Calculator** and find answer up to 3 decimal places] Approximate the area under the curve $y = x^3 + e^{\sin x} + 2\cosh x$ on the interval $[-8,9]$ by using 2, 4, 7 rectangles of equal **width** with **height** as mid-point of the base of the interval of the rectangles.



$\Delta x = (9+8)/2 = 17/2$

$x_1 = -8 + \Delta x = 1/2$

$m_1 = (-8 + x_1)/2 = -3.75$

$m_2 = (x_1 + 9)/2 = 4.75$

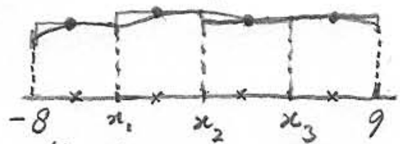
$f(m_1) = -8.4187$

$f(m_2) = 223.1330$

Area of 2 Rectangles

$= \Delta x\,[f(m_1) + f(m_2)]$

$\approx \boxed{1825.1}$



$\Delta x = (9+8)/4 = 17/4$

$x_1 = -3.75$; $x_2 = 0.5$ $x_3 = 4.75$

$m_1 = -5.875$; $m_2 = -1.625$; $m_3 = 2.625$, $m_4 = 6.875$

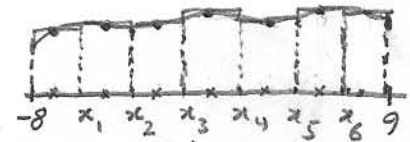
$f(m_1) =$ $f(m_2) =$

$f(m_3) =$ $f(m_4) =$

Area of 4 Rectangles

$= \Delta x\,[f(m_1) + f(m_2) + f(m_3) + f(m_4)]$

$= \boxed{6307.7}$



$\Delta x = (9+8)/7 =$

$x_1 = -5.571$; $x_2 = -3.143$; $x_3 = -0.714$

$x_4 = 1.714$; $x_5 = 4.143$; $x_6 = 6.571$

$m_1 = -6.786$; $m_2 = -4.357$; $m_3 = -1.929$

$m_4 = 0.5$; $m_5 = 2.929$; $m_6 = 5.357$; $m_7 = 7.786$

$f(m_1) = 573.28$; $f(m_2) = -2.118$; $f(m_3) = 0.244$

$f(m_4) = 3.995$ $f(m_5) = 45.107$

$f(m_6) = 366.317$; $f(m_7) = 288.6$

Area of 7 Rectangles

$= \Delta x \sum_{i=1}^{7} f(m_i)$

$= 9392.7$

2. Find the antiderivative of the following functions:

i) $5m^{-7/9} + \cot^2 m$ ii) $\dfrac{7}{4z} + \dfrac{3}{5(1+z^2)}$ iii) $(x+7)^6 - \cos^2\dfrac{x}{2}$

	i)		ii)		iii)	
Function	$m^{-7/9}$	$\cot^2 m = \csc^2 m - 1$	$1/z$	$1/(1+z^2)$	$(x+7)^6$	$\cos^2 x/2 = \frac{1}{2}(1+\cos x)$
Antiderivative	$\dfrac{m^{-7/9+1}}{-7/9+1}$	$-\cot m - m$	$\ln z$	$\tan^{-1} z$	$(x+7)^{6+1}/(6+1)$	$\frac{1}{2}(x + \sin x)$
Answer	$\frac{45}{2} m^{2/9} - \cot m - m + C$		$\frac{7}{4}\ln z + \frac{3}{5}\tan^{-1} z + C$		$\frac{(x+7)^7}{7} + \frac{1}{2}(x + \sin x) + C$	

3. Find the solution of the Differential Equation: $\dfrac{dy}{dx} = \dfrac{2}{\csc x} + x^{-2/3};\ y(\pi) = 4$

i) $\int \left(\frac{dy}{dx}\right) dx = \int \left(\frac{2}{\csc x} + x^{-2/3}\right) dx$

$y = 2(-\cos x) + 3x^{1/3} + C$

Note: $\frac{1}{\csc x} = \sin x$

$\int \frac{dx}{\csc x} = \int \sin x\, dx = -\cos x$

(ii) $x = \pi$; $y = 4$: $4 = -2\cos\pi + 3\sqrt[3]{\pi} + C \Rightarrow C = 4 + 2 - 3\sqrt[3]{\pi}$

(iii) Solution: $\boxed{y = -2\cos x + 3x^{1/3} + 6 - 3\sqrt[3]{\pi}}$

4. Evaluate:

i) $\int\left(\frac{3}{\sqrt{1-t^2}}+\frac{3}{2t\sqrt{t^2-1}}\right)dt$

$= 3\sin^{-1}t + \frac{3}{2}\sec^{-1}t + C$

Ans

ii) $\int(5^w + 7\cot w \csc w)dw$

$= \frac{5^w}{\ln 5} - 7\csc w + C$

Ans

iii) $\int \frac{1}{1-\cos 2x}dx$

$= \int \frac{1}{2\sin^2 x}dx$

$= \frac{1}{2}\int \csc^2 x\, dx$

$= \frac{1}{2}(-\cot x) + C$

$= -\frac{1}{2}\cot x + C.$

Ans

5. Suppose that a point moves along some unknown curve $y = f(x)$ in the xy-plane in such a way that at each point (x,y) on the curve, the tangent line has the slope $1+\tan^2 x$. Find an equation of the curve given that it passes through the point $\left(5, \frac{\pi}{4}\right)$. [You must read Example 6 at page 383 of the text before solving this question]

Given: ① $\frac{dy}{dx} = 1 + \tan^2 x$

(Note: $1+\tan^2 x = \sec^2 x$

$\int \sec^2 x\, dx = \tan x + C$

② $\int\left(\frac{dy}{dx}\right)dx = \int(1+\tan^2 x)\,dx$

$y = \tan x + C$

③ Curve passes through $(5, \frac{\pi}{4})$ i.e. $x = 5$, $y = \frac{\pi}{4}$

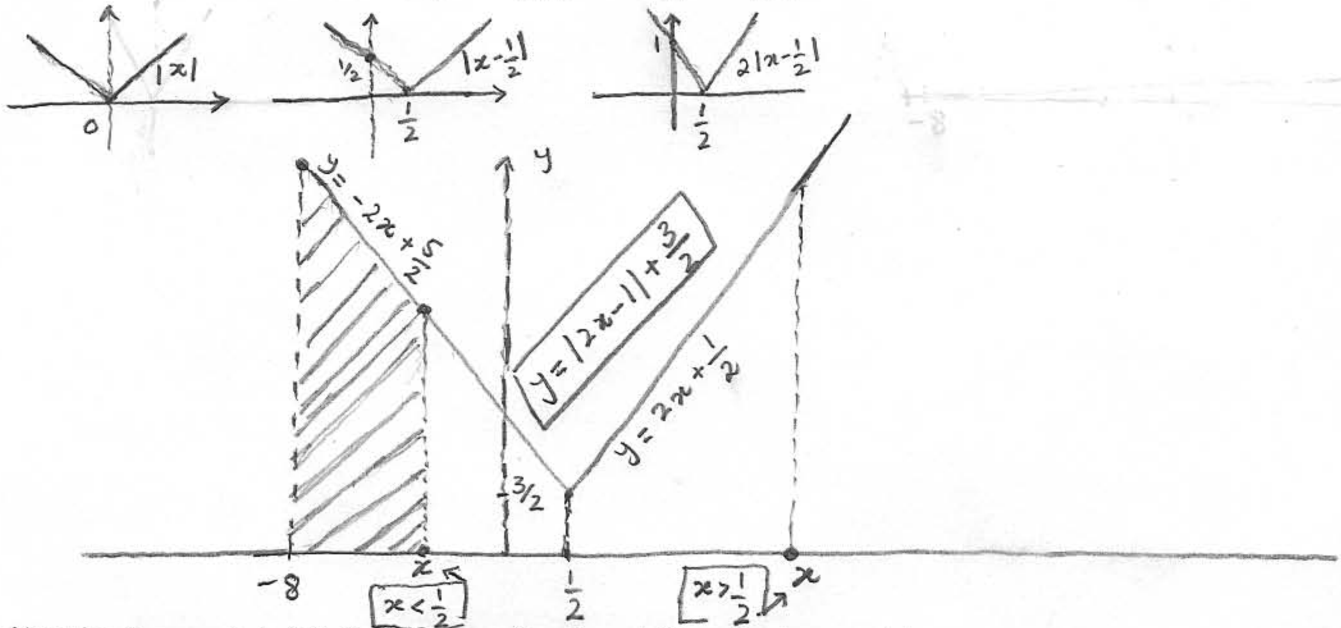
$\frac{\pi}{4} = \tan 5 + C \Rightarrow C = \frac{\pi}{4} - \tan 5$

④ Equation of the curve

$\boxed{y = \tan x + \frac{\pi}{4} - \tan 5}$

0

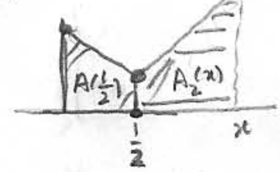
6. a) Sketch the graph of $y = |2x-1| + \frac{3}{2}$ by drawing proper xy-plane, and using **shifts** and appropriate **measurements on the axes**. Note: $|2x-1| = 2|x - \frac{1}{2}|$



b) Using the graph in (a), find the area function A(x) under the graph in the interval [-4, x].

(i) When $x < \frac{1}{2}$ then $A(x) = (x+8)\,\frac{f(-8)+f(x)}{2} = \frac{x+8}{2}(21-2x)$

(ii) When $x > \frac{1}{2}$ then $A(x) = A(\frac{1}{2}) + A_2(x)$



$$= 85 + \frac{(2x-1)(x+1)}{2}$$

$A(\frac{1}{2}) = \frac{\frac{1}{2}+8}{2}(21-1)$
$= \frac{17}{2}(10) = 85$

$A_2(x) = \frac{(x-\frac{1}{2})}{2}\left(f(\tfrac{1}{2}) + f(x)\right)$
$= \frac{2x-1}{4}\left(\frac{3}{2} + 2x + \frac{1}{2}\right)$
$= \frac{2x-1}{4}(2x+2)$
$= (2x-1)(x+1)/2$

ie.

$$A(x) = \begin{cases} (x+8)\left(\frac{21-2x}{2}\right) & ; \quad x \le \frac{1}{2} \\ (2x-1)(x+1)/2 & ; \quad x > \frac{1}{2} \end{cases}$$

c) Find the Derivative of the Area Function.

i- $x < \frac{1}{2}$: $A'(x) = \frac{1}{2}\left(21 - 2x + (x+8)(-2)\right) = -2x + \frac{5}{2} = -2x + 1 + \frac{3}{2}$

ii- $x > \frac{1}{2}$: $A'(x) = \frac{1}{2}\left(2x - 1 + (x+1)(2)\right) = 2x + \frac{1}{2} = 2x - 1 + \frac{3}{2}$

i.e., $A'(x) = |2x-1| + \frac{3}{2}$ when $x \ne \frac{1}{2}$

d) Is A(x) differentiable for all values of x.? Explain.

$A(x)$ is not differentiable for $x = \frac{1}{2}$ since $A(x)$ has a corner point at $x = \frac{1}{2}$.