Name Solution **MATH 102 - Quiz 4** **D. #**__________

(1) Show that the following series is absolutely Convergent and hence convergent $\sum_{k=1}^{\infty} \frac{7(-4)^{k+2}}{3^{2k+1}}$

Using the ratio test for absolute conv.

$$\lim_{k\to\infty} \left|\frac{a_{k+1}}{a_k}\right| = \lim_{k\to\infty} \left| \frac{4^{k+3}}{3^{2k+3}} \cdot \frac{3^{2k+1}}{4^{k+2}} \right|$$

$$= \lim_{k\to\infty} \left|\frac{4}{3^2}\right|$$

$$= \frac{4}{9} < 1$$

$\therefore$ the series is absolutely convergent.

(2) Determine whether the following series Converges absolutely, converges conditionally, or diverges $\sum_{k=1}^{\infty} \frac{(-1)^k}{5k+4}$

For $\sum_{k=1}^{\infty} \left|\frac{(-1)^k}{5k+4}\right| = \sum_{k=1}^{\infty} \frac{1}{5k+4}$,

Consider $\sum_{k=1}^{\infty} \frac{1}{k}$ which is div. [harmonic]

By the limit comparison test with $b_k = \frac{1}{k}$, we have $\lim_{k\to\infty} \frac{a_k}{b_k} = \lim_{k\to\infty} \frac{\frac{1}{5k+4}}{\frac{1}{k}} = \frac{1}{5}$. Hence $\sum_{k=1}^{\infty} \frac{1}{5k+4}$ div.

However, using the alternating series test for $\sum_{k=1}^{\infty} \frac{(-1)^k}{5k+4}$

we can see (i) $a_k > a_{k+1}$ (ii) $\lim_{k\to\infty} a_k = 0$ $\Big\} \Rightarrow \sum_{k=1}^{\infty} \frac{(-1)^k}{5k+4}$

This shows that the given series is Conditionally conv.

(3) Show that the series $\sum_{k=1}^{\infty} \frac{\sin k}{k^3}$ converges

$$|\sin k| \leq 1$$

$$\frac{|\sin k|}{k^3} \leq \frac{1}{k^3}$$

$\sum_{k=1}^{\infty} \frac{1}{k^3}$ Conv. [Power series; $p = 3 > 1$]

Hence by the Comparison test, the given series converges.

(4)(a) Show that the series $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{k}{5^k}$ satisfies the conditions of the alternating series test and hence converges.

(b) Find an upper bound on the magnitude of the error that results if the sum of the series is approximated by S_8.

(a) $a_k = \frac{k}{5^k}$

$\Rightarrow$ (i) $a_k > a_{k+1}$ since $\frac{a_k}{a_{k+1}} = \frac{k}{5^k} \cdot \frac{5^{k+1}}{k+1} = \frac{5k}{k+1} > 1$

(ii) $\lim_{k\to\infty} a_k = \lim_{k\to\infty} \frac{k}{5^k} = 0$ [how ?]

Hence the conditions of the alternating test are satisfied and the series Converges.

(b) $|\text{error}| = |S - S_8| < a_9$

$\therefore$ an upper bound of the error is a_9, where

$$a_9 = \frac{9}{5^9}$$