Name________________ **MATH 102 - Quiz 5a** **I.D. #**__________

(1) Find the interval and radius of convergence for the series $\sum_{k=1}^{\infty}(-1)^k \frac{x^k}{k}$

$$\lim_{k\to\infty}\left|\frac{a_{k+1}}{a_k}\right| = \lim_{k\to\infty}\left|\frac{x^{k+1}}{k+1}\cdot\frac{k}{x^k}\right| = \lim_{k\to\infty}\left|\frac{k}{k+1}x\right| = |x| .$$

The Series converges for $|x|<1$ i.e. $-1<x<1$. we check for the endpoints:

when $x=1 \Rightarrow \sum_{k=1}^{\infty}\frac{(-1)^k}{k}$ which is a convergent series by Alternating Series Test.

when $x=-1 \Rightarrow \sum_{k=1}^{\infty}\frac{1}{k}$ which is a divergent series (harmonic).

$\therefore$ The interval of Convergence $= (-1,1]$. The radius $R=1$.

(2) Write the first nonzero 4 terms of the Maclaurin series for $\frac{e^x}{1-x}$

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \cdots$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots$$

$$\frac{e^x}{1-x} = e^x\cdot\frac{1}{1-x} = \left(1+x+\frac{x^2}{2}+\frac{x^3}{3!}+\cdots\right)\left(1+x+x^2+x^3+\cdots\right)$$
$$= 1+2x+\frac{5}{2}x^2+\frac{8}{3}x^3+\cdots$$

(3) Find the Maclaurin series of $\sin 2x$

$f(x) = \sin 2x \rightarrow f(0)=0$

$f'(x) = 2\cos 2x \longrightarrow$

$f''(x) = -4\sin 2x = -2^2\sin 2x$

$f'''(x) = -8\cos 2x = -2^3\cos 2x$

$f^{(4)}(x) = 2^4\sin 2x$

$f^{(5)}(x) = 2^5\cos 2x$

$\vdots$

$$\left[\begin{array}{l} f'(0) = 2 \\ f''(0) = 0 \\ f'''(0) = -2^3 \\ f^{(4)}(0) = 0 \\ f^{(5)}(0) = 2^5 \\ \vdots \end{array}\right] \Rightarrow \left\{\begin{array}{l} f^{(2k)}(0) = 0 \\ f^{(2k+1)}(0) = (-1)^k 2^{2k+1} \end{array}\right\} \quad (*)$$

$$\therefore f(x) = \sin 2x = \sum_{k=0}^{\infty}\frac{f^{(k)}(0)}{k!}x^k$$
$$= \sum_{k=0}^{\infty}(-1)^k\frac{2^{2k+1}}{(2k+1)!}x^{2k+1}, \quad \text{by } (*).$$