Math 202 Quiz # 4

Name: Solution I.D. # _____ Section # ____

Given $y_1 = x$ is a solution of the differential equation $(x^2+1)y'' - 2xy' + 2y = 0$, find a second linearly independent solution.

Note that you may assume that $y_2 = u(x)y_1$ and proceed as we did in the class.

Alternatively, you can use the formula to obtain another solution y_2 as follows.

First we put the DE in the standard form:

$$y'' - \frac{2x}{x^2+1}y' + \frac{2}{x^2+1}y = 0$$

$$y_2 = y_1 \int \frac{e^{-\int p(x)\,dx}}{y_1^2}\,dx, \quad \text{where } p(x) = \frac{-2x}{x^2+1}$$

$$y_2 = x \int \frac{e^{\int \frac{2x}{x^2+1}\,dx}}{x^2}\,dx$$

$$= x \int \frac{e^{\ln(x^2+1)}}{x^2}\,dx$$

$$= x \int \frac{x^2+1}{x^2}\,dx \qquad = x \int \left(1 + \frac{1}{x^2}\right) dx$$

$$= x\left(x - \frac{1}{x}\right)$$

$$\therefore y_2 = x^2 - 1$$