Questions from Old Exams

1 Section 7.3

1. Find the smallest positive angle between two vectors $v = \langle 1, 1\rangle$, and $w = \frac{-3}{\sqrt{2}}i - \frac{3}{\sqrt{2}}j$.

2. Given $v = 6i - 4j$, $u = 2i + 3j$, and $w = \langle 2, -2\rangle$. Find the scalar projection of $\frac{1}{2}v - 3u$ on w.

3. Given the vectors $U = \langle\sqrt{3}, 1\rangle$ and $V = \langle 1, \sqrt{3}\rangle$, find

   (a) The dot product of the vectors U and V.

   (b) The angle between the vectors U and V.

   (c) A vector W of magnitude 6 in the opposite direction of the vector $U - \sqrt{3}V$.

4. Which one of the following statements is always TRUE for any two nonzero vectors u and v and any nonzero real number k?

   (a) The vector $\frac{-u}{||u||}$ is a unit vector.

   (b) The vectors u and ku have the same direction.

   (c) $||u + v|| < ||u|| + ||v||$.

   (d) $||kv|| = k\,||v||$.

5. Given the vectors $u = -2i + 3j$ and $v = i + 5j$.

   (a) Find a vector of length 3 in the opposite direction of the vector u.

   (b) Find the measure of the smallest angle between the vectors u and v.

6. Suppose that the vector $u = \overrightarrow{PQ}$, where the initial point is $P(5, 4)$, and the terminal point is $Q(5, 11)$. If $v = \sqrt{3}i - 8j$, then find the magnitude and the direction angle of the vector $u + v$.

7. If $\overrightarrow{u} = \langle 2\sqrt{3}, -3\rangle$ and $\overrightarrow{v} = \langle -\sqrt{3}, 4\rangle$ then find the magnitude and the direction angle of $\overrightarrow{u} + \overrightarrow{v}$.

8. Find the direction angle α and the magnitude $||v||$ of the vector $v = \sqrt{3}i - 3j$.

9. Find all values of the constant k for which the two vectors $u = (k - 1)\,i + j$ and $v = 3i + (k - 1)^2\,j$ are perpendicular.

10. Given the vectors $u = \langle 3, -5\rangle$, and $v = \langle -3, -1\rangle$. Find the magnitude and the direction angle of the vector $u + v$.

11. If $\overrightarrow{v} = \langle -2, -6\sqrt{3} \rangle$ and $v = -\sqrt{3}i - j$, then find the magnitude and the direction angle of the vector $\frac{\overrightarrow{v}}{2} - \sqrt{3}\overrightarrow{u}$.

12. If θ, $0 \leq \theta \leq \pi$, is the angle between the vectors $\overrightarrow{u} = -i + 2j$ and $\overrightarrow{v} = 2i - j$, then find $\sin\theta$.

13. Find a unit vector in the opposite direction of the vector $\overrightarrow{v} = \langle -2, 3 \rangle$.

14. If $\overrightarrow{u} = \langle 1, 2 \rangle$ and $\overrightarrow{v} = \langle -3, \sqrt{3} - 1 \rangle$, then find the magnitude and the direction angle of the vector $\overrightarrow{u} + \overrightarrow{v} + i - j$.

15. Given the vectors $\overrightarrow{u} = \langle 3, -4 \rangle$, $\overrightarrow{v} = \langle 4, 3 \rangle$, and $\overrightarrow{w} = \langle a, b \rangle$. If $\overrightarrow{w}$ is a unit vector opposite in direction to the vector $\overrightarrow{u} + \overrightarrow{v}$, then find the value of a and b.

16. Find the smallest angle between the vectors $\overrightarrow{u} = -i - 2j$ and $\overrightarrow{v} = -i + 3j$.

17. If $\overrightarrow{u} = \langle 2\cos 80°, 2\sin 80° \rangle$ and $\overrightarrow{v} = \langle 3\cos 20°, 3\sin 20° \rangle$, then find $\overrightarrow{u} \cdot \overrightarrow{v}$.

18. If t is the angle between the unit vectors $\overrightarrow{u}$ and $\overrightarrow{v}$ and $\cos t = \frac{1}{3}$, then find $||\overrightarrow{u} - \overrightarrow{v}||$.

19. For the vectors $\overrightarrow{u} = \langle 4, 3 \rangle$ and $\overrightarrow{v} = \langle 2, 1 \rangle$, let α be the angle between $\overrightarrow{u}$ and $\overrightarrow{v}$, where $0° \leq \alpha \leq 180°$, then find the value of $(\overrightarrow{u} - \overrightarrow{v}) \cdot (\overrightarrow{u} + \overrightarrow{v}) + 2\,||\overrightarrow{u}||\,||\overrightarrow{v}||\cos\alpha$.

20. If $\overrightarrow{u}$ and $\overrightarrow{v}$ are unit vectors such that $\overrightarrow{u} \cdot \overrightarrow{v} = -\frac{3}{4}$, then find $||\overrightarrow{u} + \overrightarrow{v}||$.

21. Find two unit vectors which are perpendicular to the vector $\overrightarrow{v} = i + 2j$.

22. If $\overrightarrow{u} = \langle 2, 1 \rangle$ and $\overrightarrow{v} = \langle 1, -2 \rangle$, then $\overrightarrow{u} + \overrightarrow{v}$ is perpendicular to $a) \overrightarrow{v}$ $b) \overrightarrow{u}$ $c) \overrightarrow{u} + \overrightarrow{v}$ $d) \overrightarrow{u} - \overrightarrow{v}$ $e) \overrightarrow{u} - 2\overrightarrow{v}$

23. If θ is the angle between $\overrightarrow{u} = i + 3j$ and $\overrightarrow{v} = -i + 3j$, and $0° \leq \theta \leq \pi$, then find $\tan\theta$.

24. If $\overrightarrow{u}$ and $\overrightarrow{v}$ are unit vectors and the angle between $\overrightarrow{u}$ and $\overrightarrow{v}$ is $60°$, then find the magnitude of the vector $2\overrightarrow{u} - 3\overrightarrow{v}$.

25. For vectors $\overrightarrow{u}$ and $\overrightarrow{v}$, if $||\overrightarrow{u} + \overrightarrow{v}|| = \sqrt{50}$ and $||\overrightarrow{u}|| = ||\overrightarrow{v}|| = 4$, then find $\overrightarrow{u} \cdot \overrightarrow{v}$.

26. Which one of the following is not a unit vector?

    (a) $\langle \frac{1}{2}, -\frac{\sqrt{3}}{2} \rangle$
    (b) $\langle -1, 0 \rangle$
    (c) $\langle \sin\frac{\pi}{5}, \cos\frac{\pi}{5} \rangle$
    (d) $\langle \frac{1}{\sqrt{3}}, \frac{\sqrt{2}}{\sqrt{3}} \rangle$
    (e) $\langle \frac{1}{5}, \frac{4}{5} \rangle$

27. Find the vector of magnitude 2 and opposite to $\overrightarrow{u} = \langle -1, 1 \rangle$.

28. If $||\overrightarrow{u}|| = 2$ with direction angle $\theta = 30°$, then find the vector $\overrightarrow{u}$.

29. Find the magnitude and the direction angle of $\overrightarrow{u} = 2\langle 6, -1 \rangle - 3\langle 4, 5 \rangle$.

30. If $\overrightarrow{w}$ has magnitude 24 and direction angle 30°, then find the horizontal and vertical components of $\overrightarrow{w}$.

31. If θ is the direction angle of $\overrightarrow{u} = \langle -3, 4 \rangle$, then find $\tan 2\theta$.

32. If $\overrightarrow{u}$ and $\overrightarrow{v}$ are two nonzero perpendicular vectors, then

    (a) $||\overrightarrow{u} + \overrightarrow{v}|| > ||\overrightarrow{u} - \overrightarrow{v}||$
    (b) $||\overrightarrow{u} - \overrightarrow{v}|| > ||\overrightarrow{u} + \overrightarrow{v}||$
    (c) $||\overrightarrow{u} + \overrightarrow{v}|| = 0$
    (d) $||\overrightarrow{u} + \overrightarrow{v}|| = ||\overrightarrow{u} - \overrightarrow{v}||$
    (e) $||\overrightarrow{u} - \overrightarrow{v}|| = 0$

33. If $\overrightarrow{u} = 4i + j$, $\overrightarrow{v} = \langle 1, 3 \rangle$, and $\overrightarrow{w} = \overrightarrow{u} - \overrightarrow{v} - 2j$, then

    (a) $\langle -1, 1 \rangle$ is a unit vector opposite to $\overrightarrow{w}$.
    (b) $\langle 1, -1 \rangle$ is a unit vector opposite to $\overrightarrow{w}$.
    (c) $\langle -\frac{3}{5}, \frac{4}{5} \rangle$ is a unit vector opposite to $\overrightarrow{w}$.
    (d) $\langle 3, 4 \rangle$ is a unit vector in the direction of $\overrightarrow{w}$.
    (e) The direction angle of $\overrightarrow{u}$ is greater than the direction angle of $\overrightarrow{v}$.

34. If $\overrightarrow{u} = \langle 2, -1 \rangle$ and $\overrightarrow{v} = -3i + 2j$, then find the direction angle of the vector $2\overrightarrow{u} + \overrightarrow{v} + i - 2\sqrt{3}j$.