Questions from Old Exams

1 Section 10.1

1. Find the echelon form of the matrix $\begin{bmatrix} 1 & -3 & 2 & -4 \\ 2 & 0 & -2 & 4 \\ 0 & 4 & 2 & 11 \end{bmatrix}$.

2. Consider the augmented matrix of a linear system $\begin{bmatrix} 1 & -2 & -2 & M & -1 \\ 1 & 1 & 1 & M & 2 \\ 1 & 2 & 2 & M & 1 \end{bmatrix}$.

   Which one of the following statements is TRUE?

   (a) The system is independent.
   (b) The system is dependent.
   (c) The system has the solution $\left\{\left(2, 1, \frac{1}{2}\right)\right\}$.
   (d) The system has the solution $\{(5, -1, -1)\}$.
   (e) The system has no solution.

3. Which one of the following statements is TRUE about the linear system of equations which has the augmented matrix $\begin{bmatrix} 1 & 2 & -1 & 1 \\ 2 & 4 & -2 & 0 \\ 1 & 2 & (c-1)^2 & c+1 \end{bmatrix}$.

   (a) The system is consistent if $c = 0$, with infinitely many solutions.
   (b) The system is consistent for all $c \neq 0$, with exactly one solution.
   (c) The system can be made consistent for suitable choice of c.
   (d) The system is inconsistent for all values of c.
   (e) The system is consistent for $c > 0$.

4. If the augmented matrix of a system of linear equations is $\begin{bmatrix} 1 & 2 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 1 & 2 & 3 & -2 \\ 0 & 0 & 0 & 3 & -3 \end{bmatrix}$,

   then find the solution set of the system.

5. If the augmented matrix of a system of linear equations is $\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & 1 & 2 & 3 & 4 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}$

   then find the solution set of the system.

6. If the augmented matrix of a system of linear equations is $\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 0 & 0 & 1 \\ 0 & 0 & 2 & 1 & 0 \\ 0 & 0 & 4 & 2 & 2 \end{bmatrix}$

   then

(a) the system has infinitely many solutions.
(b) the system has a unique solution.
(c) the matrix can not be the augmented matrix of a 4×4 system.
(d) the system has two solutions.
(e) the system has no solution.

7. Which one of the following represents an inconsistent system?

(a) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 0 & 3 & 1 \\ 0 & 1 & 2 & 3 \\ 0 & 2 & 4 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 2 & 0 & 2 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

(e) $\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 1 & 1 & 1 & 0 \end{bmatrix}$

8. If the augmented matrix of a system of linear equations is $\begin{bmatrix} -1 & 1 & 0 & -1 \\ 0 & 1 & -1 & 6 \\ 1 & 0 & 1 & -1 \end{bmatrix}$, then find the solution set of the system.

9. If the system $\begin{bmatrix} 1 & 1 & 1 & 2 \\ 3 & 2 & 4 & 5 \\ 2 & 1 & 1 & 6 \end{bmatrix}$ is written as $\begin{bmatrix} 1 & m & n & 2 \\ 0 & 1 & k & 1 \\ 0 & 0 & 1 & -\frac{3}{2} \end{bmatrix}$, then find mnk.

10. Find the solution set of $\begin{bmatrix} 1 & 1 & 1 & -3 \\ 2 & -1 & 1 & 1 \\ 4 & 1 & 3 & 5 \end{bmatrix}$.

11. If (a, b, c) is the solution for $\begin{bmatrix} 1 & 2 & -1 & 5 \\ 2 & -1 & 3 & 0 \\ 1 & 1 & 1 & 2 \end{bmatrix}$, then find the value of $3a + 4b + c$.

12. Given the system $\begin{bmatrix} 1 & -2 & 4 & 2 \\ 0 & 1 & 3 & -1 \\ 0 & 2 & 6 & A \end{bmatrix}$. Which one of the following is FALSE?

(a) The system is inconsistent for all $A \neq 2$.

(b) The system is consistent with infinitely many solutions for $A = -2$.

(c) The system has no unique solution for any real number A.

(d) The system can be made consistent or inconsistent for a suitable choice of A.

(e) The system is consistent for any real A.

2 Section 10.2

1. If $C = AB$ where $A = \begin{bmatrix} 1 & -1 & 0 & 1 \\ 2 & 3 & -1 & 4 \\ -1 & 2 & 1 & 3 \\ 0 & 1 & -1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 & 0 & 1 \\ 1 & -1 & 2 & -1 \\ 0 & 1 & 1 & 1 \\ 1 & -1 & 0 & 2 \end{bmatrix}$, then find C_{23}, the third row and second column of C.

2. Given the matrices $A = \begin{bmatrix} 3 & 2 & 0 \\ 3 & 5 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 5 & 0 \\ -3 & 1 \\ 0 & -1 \end{bmatrix}$, and $C = \begin{bmatrix} \frac{3}{2} & 1 \\ 0 & \frac{3}{2} \end{bmatrix}$, then find the matrix $AB - 2C$.

3. If $A = \begin{bmatrix} 0 & -2 & 7 \\ 5 & 4 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 1 \\ -1 & 5 \\ 6 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 40 & -10 \\ 28 & 23 \end{bmatrix}$, and $D = AB - C$, then find the element in the second row and second column of the matrix D.

4. Given $A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 0 \\ x & 2 \end{bmatrix}$, and $C = \begin{bmatrix} 0 & 0 \\ 6 & 0 \end{bmatrix}$. If $AB = 2A^2 - C$, then find x.

5. Let $A = \begin{bmatrix} 2 & -3 \\ 0 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 3 \\ 0 & 1 \end{bmatrix}$. If X is a 2×2 matrix such that $X = 2A - B$, then

   (a) $X = -3B$

   (b) $X = 2A$

   (c) $X = -2B$

   (d) $X = 2B$

   (e) $X = -3B$

6. If $A = \begin{bmatrix} 1 & 4 \\ 0 & -1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 2 \\ 3 & 4 & 0 \end{bmatrix}$, then find the element a_{32} of A.

7. Let $A = \begin{bmatrix} -1 & 2 \\ 3 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$, and $C = \begin{bmatrix} \mathrm{a} & \frac{1}{2} \\ 3 & b \end{bmatrix}$. If $AB = 2C$, then find a and b.

8. If $A = \begin{bmatrix} 1 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$ and $B = \begin{bmatrix} \frac{1}{2} & 1 \\ \frac{1}{2} & 0 \end{bmatrix}$, then

   (a) $A + B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

   (b) $A - B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

   (c) $AB = BA$

   (d) $AB = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

   (e) $AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

9. If $A = \begin{bmatrix} -1 & 0 & 1 \\ 3 & 1 & -2 \\ 0 & -2 & 0 \end{bmatrix}$, then find the element in the second row and third column of $\left(A^2 - A\right)$.

10. If $C = \begin{bmatrix} 2 & 2 & 1 & 1 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & -1 & 0 \end{bmatrix}$ and $D = \begin{bmatrix} 1 & 2 & -1 & 0 \\ 0 & 0 & 2 & 1 \\ 4 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}$, then find the element in the second row and third column of CD.

11. If C is 4×3, A and B are 3×4, then find the size of $C \cdot (2A + 3B)$.

12. If A and B are two matrices of size 4×3, then find the size of $B^T \cdot (2A + 3B)$.

13. If $A = \begin{bmatrix} 2 & 1 \\ 0 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 1 \\ 4 & 5 \end{bmatrix}$, and $C = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$, then find $A^T - 2B + C^2$.

14. If $A = \begin{bmatrix} -1 & 0 \\ 3 & 1 \\ 0 & -2 \end{bmatrix}$, and $B = \begin{bmatrix} 2 & 1 \\ 1 & 0 \\ -1 & 2 \end{bmatrix}$, then find $(A + B) \cdot B^T$.

15. Let A and B be square matrices of the same order and A^T is the transpose of A. Which one of the following is not always true?

   (a) $\left(A^T\right)^T = A$

   (b) $(A + B)^T = A^T + B^T$

(c) $(A+B)^2 = A^2 + 2AB + B^2$.

(d) $(AB)^T = B^T A^T$

(e) $c\,(A+B) = cA + cB$, where c is a real number.

16. If $A = \begin{bmatrix} 3 & 2 \\ x & 0 \\ -2 & -1 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 2 \\ 3 & 0 \end{bmatrix}$ and $B^T A^T = 2 \begin{bmatrix} \frac{3}{2} & \frac{1}{2} & \frac{x}{2} \\ y & -1 & -2 \end{bmatrix}$, then find x and y.

3 Section 10.3

1. If $AX = B$ is the matrix equation which represents the system $\begin{cases} 3x + 2y = 1 \\ 2x + y = 6 \end{cases}$, then find X.

2. Given the matrices $A = \begin{bmatrix} 2 & -1 \\ 4 & -3 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 \\ 3 & -5 \end{bmatrix}$, $C = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$. If $(A-B)\,X = C$, then $X =$

   (a) $\begin{bmatrix} 2 & 2 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 4 \\ 2 \end{bmatrix}$

   (b) $\begin{bmatrix} 2 & 2 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

   (c) $\begin{bmatrix} 0 & -1 \\ \frac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 2 \end{bmatrix}$

   (d) $\begin{bmatrix} 2 \\ 1 \end{bmatrix} \begin{bmatrix} 2 & 2 \\ 1 & 0 \end{bmatrix}$

   (e) $\begin{bmatrix} 4 \\ 2 \end{bmatrix} \begin{bmatrix} 2 & 2 \\ 1 & 0 \end{bmatrix}$

3. If $A^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is the inverse of $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$, then find a and c.

4. If the matrix $\begin{bmatrix} 2 & 4 & 3 \\ 0 & 1 & -1 \\ 3 & 5 & 7 \end{bmatrix}$ is the multiplication inverse of $\begin{bmatrix} 4 & -13t & -7t \\ x & 5t & yt \\ -1 & 2t & 2t \end{bmatrix}$, then find x, y, and t.

5. If $a \neq 0$ and $A = \begin{bmatrix} a & 0 & 0 \\ 0 & 2 & -4 \\ 0 & 1 & -2 \end{bmatrix}$, then

   (a) $AA^{-1} = I$

   (b) $A^{-1}A = I$

(c) $A^{-1} = \begin{bmatrix} a^{-1} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} & 1 \end{bmatrix}$

(d) A^{-1} does not exist.

(e) $A^{-1} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

6. If A and B are matrices of order $n \times n$, then

   (a) if A^{-1} exists, then $ABA^{-1} = B$ where $B \neq I$.

   (b) A^{-1} and B^{-1} are $(n+1) \times (n+1)$ matrices.

   (c) if $AB = O$, then either $A = O$ or $B = O$, where O is an $n \times n$ zero matrix.

   (d) if A^{-1} exists, then $\left(AA^{-1}\right)$ is the $n \times n$ identity matrix.

   (e) $(A + 2B)(A - 2B) = A^2 - 4B^2$.

7. If $A^{-1} = \begin{bmatrix} 2 & -1 & 1 \\ -3 & 0 & 1 \\ 0 & 2 & 2 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} 0 & -1 & 0 \\ 3 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$, then find the element in the second row and third column of $(AB)^{-1}$.

8. If $A = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 3 \\ 2 & 2 \end{bmatrix}$, and $O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, then

   (a) $A^{-1} = -A$, B^{-1} does not exist.

   (b) $A^{-1} = A$, B^{-1} does not exist.

   (c) $A^{-1} = A$, $B^{-1} = O$.

   (d) A^{-1} does not exist, $B^{-1} = O$.

   (e) $A^{-1} = \begin{bmatrix} \frac{2}{7} & \frac{1}{7} \\ \frac{-3}{7} & \frac{-2}{7} \end{bmatrix}$, B^{-1} does not exist.

9. If $A^{-1} = \begin{bmatrix} 2 & 0 & 1 \\ 3 & 0 & x \\ -6 & 1 & -4 \end{bmatrix}$ is the inverse of $\begin{bmatrix} 2 & -1 & 0 \\ 0 & 2 & 1 \\ -3 & 2 & 0 \end{bmatrix}$, then find x.

10. Given the system $\begin{cases} 3x - 2y = 4 \\ x + 3y = 5 \end{cases}$. Find the element in the first row and first column of the inverse of the coefficient matrix.

11. If $A = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \\ -1 & 1 & 1 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} x & 3x & -2x \\ 2x & 0 & 2x \\ -x & 3x & 2x \end{bmatrix}$, which of the following is TRUE?

(a) $x < -\frac{1}{2}$
(b) $\frac{1}{2} < x < 1$
(c) $x > 1$
(d) $-\frac{1}{2} < x < \frac{1}{2}$
(e) $1 < x < 2$

12. If A and B are $n \times n$ matrices and A^{-1} and B^{-1} exist, then which one of the following is not always true?

    (a) $(AB)^T = B^T A^T$
    (b) A^{-1} is $n \times n$.
    (c) $AA^{-1} = I$
    (d) $(AB)^{-1} = A^{-1}B^{-1}$
    (e) $(A+B)^2 = A^2 + B^2 + AB + BA$

13. If $\begin{bmatrix} -2 & -3 \\ -3 & -4 \end{bmatrix}$ is the inverse of $\begin{bmatrix} 4 & n \\ m & 2 \end{bmatrix}$, then find m and n.

14. If the matrix equation $A^3 = I$ is true and A^{-1} exists, then $A^{-1} =$ $a)A^2$ $b)A$ $c)A^3$ $d)I$ $e)A^6$

15. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$, then find the element in the second row and the third column of A^{-1}.

16. Given the matrix equation $AXC = B$. If A^{-1} and C^{-1} exist, then $X =$

    (a) $A^{-1}BC^{-1}$
    (b) $BA^{-1}C^{-1}$
    (c) $A^{-1}C^{-1}B$
    (d) $BC^{-1}A^{-1}$
    (e) $C^{-1}BA^{-1}$

17. The solution set of the matrix equation $\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ is given by:

    (a) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$
    (b) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$
    (c) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -\frac{2}{7} & \frac{3}{7} \\ \frac{1}{7} & \frac{-2}{7} \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(d) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \begin{bmatrix} -\frac{2}{7} & \frac{3}{7} \\ \frac{1}{7} & \frac{-2}{7} \end{bmatrix}$

(e) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 1 \\ \frac{1}{3} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

18. If $A = \begin{bmatrix} 1 & 1 & 4 \\ 2 & 3 & 6 \\ -1 & -1 & 2 \end{bmatrix}$, then find the sum of the elements in the second row of A^{-1}.

19. Suppose that $A = \begin{bmatrix} 3 & 2 \\ 2 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 22 \\ 10 \end{bmatrix}$, and $X = \begin{bmatrix} x \\ y \end{bmatrix}$. If $AX = B$, then the matrix X is equal to:

(a) $\begin{bmatrix} 2 & 2 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} 22 \\ 10 \end{bmatrix}$

(b) $\begin{bmatrix} -2 & 2 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} 22 \\ 10 \end{bmatrix}$

(c) $\begin{bmatrix} 11 \\ 5 \end{bmatrix} \begin{bmatrix} 2 & -2 \\ -2 & 3 \end{bmatrix}$

(d) $\begin{bmatrix} 2 & -2 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} 11 \\ 5 \end{bmatrix}$

(e) $\begin{bmatrix} 22 \\ 10 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & \frac{3}{2} \end{bmatrix}$

20. Which of the following is TRUE for square matrices A and B which are the same size?

(a) If $AB = O$, then $A = O$ or $B = O$.
(b) $(A+B)^2 = A^2 + 2AB + B^2$
(c) $(A-B)(A+B) = A^2 - B^2$
(d) $(AB)^{-1} = B^{-1}A^{-1}$
(e) $A(BC) = (BA)C$

21. If $A = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, then find the element in row 2 column 3 of A^{-1}.

22. If the matrix $M$and its inverse are given by $M = \begin{bmatrix} 2 & 2 & -1 \\ 0 & 3 & -1 \\ -1 & -2 & 1 \end{bmatrix}$,

$M^{-1} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 2 \\ x & y & z \end{bmatrix}$, then find $x + y + z$.

4 Section 10.4

1. If A and B are two matrices of order 4 such that $|A| = 4$ and $|B| = 5$, then find the value of $|AB| - 5\left|B^{-1}\right|$.

2. Find the value of the determinant $\begin{vmatrix} 2 & 0 & 0 & 1 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 1 & 0 & 0 & 1 \end{vmatrix}$.

3. Find the minor and the cofactor of the element 0 in the matrix $\begin{bmatrix} -3 & 2 & 1 \\ -5 & 6 & 0 \\ -2 & -1 & 3 \end{bmatrix}$.

4. If A and B are two matrices of order 3×3 and $|A| = 4$ and $|B| = 5$, then find the value of $2|A| - \left|2B^{-1}\right|$.

5. Find the value of the determinant $\begin{vmatrix} 4 & -1 & 3 \\ 3 & 1 & 2 \\ 1 & -1 & 1 \end{vmatrix}$.

6. If $M = \begin{bmatrix} 5 & 6 \\ 4 & 0 \end{bmatrix}$ and I is the 2×2 identity matrix, then find the sum of the values of x which satisfy $\det(M - xI) = 0$.

7. If Z is a 5×5 matrix and $|Z| = 3$, then find $\left|2Z^{-1}\right|$.

8. Find the value of the determinant $\begin{vmatrix} -1 & 2 & 2 & 3 \\ 0 & 2 & 3 & 4 \\ 0 & 2 & 6 & 6 \\ 2 & -4 & -4 & -2 \end{vmatrix}$.

9. If A is a square matrix with inverse A^{-1} and transpose A^T, then which one of the following is always TRUE?

   (a) $\left|A^T\right| = -|A|$

   (b) $\left|AA^{-1}\right| = 1$

   (c) $\left|AA^T\right| = 1$

   (d) $\left|A^{-1}\right| = |A|$

   (e) $\left|A^{-1}\right| = \left|A^T\right|$

10. If $\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ 2 & 3 & 4 \end{vmatrix} = 3$, then find $\begin{vmatrix} 2 & 3 & 4 \\ x-4 & y-6 & z-8 \\ -2 & -2 & -2 \end{vmatrix}$.

11. Find the solution set of $\begin{vmatrix} x & x & 0 \\ 2 & 1+x & 2 \\ -1 & 0 & x \end{vmatrix} = 0$.

12. Find the value of $\begin{vmatrix} a & b & 0 & 0 \\ c & d & 0 & 0 \\ 0 & 0 & a & b \\ 0 & 0 & c & d \end{vmatrix}$.

13. If $A = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$, $B = \begin{vmatrix} c & -2b & -3a \\ f & -2e & -3d \\ i & -2h & -3g \end{vmatrix}$ and $|A| = 2$, then find $|B|$.

14. Find the determinant $\begin{vmatrix} x & 2 & 4 \\ 1 & 3 & 0 \\ 1-2x & -1 & -8 \end{vmatrix}$.

15. If a 3×3 matrix A with elements a_{ij} has $a_{11} = -1$, $a_{21} = 3$, and $a_{31} = 4$, and the minors of a_{11}, a_{21}, and a_{31} are 5, -2, 3 respectively, then find $|A|$.

16. Find the determinant $\begin{vmatrix} \sin\theta & -\cos\theta \\ -\sin 2\theta & \cos 2\theta \end{vmatrix}$.

17. Find the cofactor of x in $\begin{vmatrix} -3 & 0 & -1 & 0 \\ 2 & 4 & 6 & 2 \\ 0 & x & -2 & 4 \\ 1 & 3 & 1 & 0 \end{vmatrix}$.

18. If $A = \begin{bmatrix} 1 & 2 & 3 \\ a & b & c \\ x & y & z \end{bmatrix}$, $B = \begin{bmatrix} -1 & -2 & -6 \\ 3a & 3b & 6c \\ x & y & 2z \end{bmatrix}$, and $C = \begin{bmatrix} 2 & 4 & 6 \\ 2x & 2y & 2z \\ 2a & 2b & 2c \end{bmatrix}$, then

    (a) $B = -6A$, $C = 2A$
    (b) $|B| = -6|A|$, $|C| = -8|A|$
    (c) $|B| = -6|A|$, $|C| = -2|A|$
    (d) $|B| = 6|A|$, $|C| = 8|A|$
    (e) $|B| = -8|A|$, $|C| = -2|A|$

19. If A is a 3×3 matrix, then find $|2A|$ in terms of $|A|$.

20. Find the cofactor of he element in the third row and second column of $A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 4 \\ 1 & 2 & 6 \end{bmatrix}$.

21. If $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix}$, then find $|A|$.

22. If $B = \begin{bmatrix} 4 & 2 & -1 \end{bmatrix}$ and $A = \begin{bmatrix} 1 \\ -3 \\ 1 \end{bmatrix}$, then find $\left|A^T B^T\right|$.

23. Find the solution set of $\begin{vmatrix} 1 & 0 & 0 \\ 0 & x & 1 \\ 0 & 1 & x \end{vmatrix} = 0$.

24. Let A and B be 3×3 matrices. Which one of the following is FALSE?

    (a) $(AB)^{-1} = B^{-1}A^{-1}$.
    (b) $(|A| + 1)^2 = |A|^2 + 2|A| + 1$.
    (c) $\left|A^T\right| = |A|$
    (d) $\left|A^{-1}\right| = |A|$.
    (e) $|3A| = 27|A|$.

25. If $A = \begin{bmatrix} 0 & 1 & 2 \\ 3 & 0 & 1 \\ 1 & -1 & 1 \end{bmatrix}$, then find $M_{21}$and C_{13}.

26. Find the sum of all values of x for which $\begin{vmatrix} -1 & 3 & 0 \\ 0 & 2 & x \\ 1 & -x & 1 \end{vmatrix} = 0$.

27. If $\begin{vmatrix} 3 & x & u \\ 3 & y & v \\ 3 & z & w \end{vmatrix} = 1$, then find $\begin{vmatrix} x & z & y \\ 2 & 2 & 2 \\ u & w & v \end{vmatrix}$.

28. If A is 5×5 and $|A| = 4$, then find the value of $2|A| + \left|2A^{-1}\right|$.

29. If $A = \begin{bmatrix} 3 & -1 \\ 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$, then find $\left|(AB)^T\right|$.

30. If $\begin{vmatrix} 1 & 0 & 1 \\ 0 & \sin\theta & \cos\theta \\ \sec\theta & -\cos\theta & \sin\theta \end{vmatrix} = 0$, $0 \le \theta \le \pi$, then find θ.

31. The determinant $\begin{vmatrix} 1 & 2 & 3 \\ a & b & c \\ 2+a & 4+b & 6+c \end{vmatrix}$ is

    (a) equal to 0 only if $a = b = c = 0$.
    (b) equal to 0 only if $a = -b$ and $b = -c$.
    (c) never equal to 0.
    (d) always equal to 0.
    (e) equal to zero only if $a = -2$, $b = -4$, and $c = -6$.

32. If A is a 4×4 matrix and $|A| = \frac{3}{2}$, then find $\frac{1}{4}\left|-2A\right|$.

33. If $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$, I is the 2×2 identity matrix, then find $\left|A - 3I\right|$.

34. Find the minor M_{23} of the element x in the matrix $\begin{bmatrix} \cos 2\theta & -\sin 2\theta & 1 \\ 1 & 1 & x \\ -\sin\theta & \cos\theta & 0 \end{bmatrix}$.