

KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS  
MATHEMATICAL SCIENCES DEPARTMENT

MATH 201

EXAM # 3

Tuesday November 27, 2001

NAME:

Sec#:

ID#:

**SHOW ALL YOUR WORK**

1. a. (3pts) Show that  $\lim_{(x,y) \rightarrow (0,0)} \frac{x^3y}{2x^6 + y^2}$  does not exist.

b. (3pts each) Compute the following limits. Show all details

(i)  $\lim_{(x,y,z) \rightarrow (0,0,0)} \frac{\sin^2 \sqrt{x^2 + y^2 + z^2}}{x^2 + y^2 + z^2}$

(ii)  $\lim_{(x,y) \rightarrow (0,0)} xy \ln(x^2 + y^2)$

(iii)  $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 - xy + x - y}{y^2 - xy - 2x + 2y}$

- 2. a. (3pts)** Suppose  $z = f(u)$ ,  $u = g(x, y)$ . Draw a tree diagram and use it to find  $\frac{\partial z}{\partial x}$ ,  $\frac{\partial z}{\partial y}$  in terms of  $\frac{dz}{du}$ ,  $\frac{\partial u}{\partial x}$ ,  $\frac{\partial u}{\partial y}$ .
- b. (3pts)** Apply your results in part (a) to the case  $z = f(x^2 - y^2)$ .

- 3. (4pts)** Let  $f(x, y) = \ln \sqrt{1 + xy}$ ,  $P = (0, 2)$ ,  $Q = (-.09, 1.98)$ . Use differentials to approximate  $f$  at  $Q$ .

4. Let  $f(x, y) = \frac{xy}{x+y}$ ,  $P = (1, 0)$ ,  $Q = (-1, -1)$ .

- a. (3pts) Find the directional derivative of  $f$  at  $P$  in the direction of  $Q$ .
- b. (3pts) Find the maximum ascent of  $f$  at  $P$  and the direction in which it occurs.
- c. (2pts) Find the maximum descent of  $f$  at  $P$  and the direction in which it occurs.

5. (5pts) Show that the surfaces  $z = \sqrt{x^2 + y^2}$  and  $z = \frac{1}{10}(x^2 + y^2) + \frac{5}{2}$  intersect at  $(3, 4, 5)$  and have a common tangent plane at that point.

6. (5pts) Discuss the relative maxima, relative minima and saddle points of the function  $f(x, y) = xy + \frac{a^3}{x} + \frac{b^3}{y}$  in the cases  $a = b = 0$  and  $a > 0, b < 0$ .