KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS
DEPARTMENT OF MATHEMATICS & STATISTICS
MATH 201-05
Quiz # 1

1. Which points on $x = 3t + t^2, y = t^3 - 12$ have tangents with slope 3?

   Answer:

$$\begin{aligned} \frac{dy}{dx} &= \frac{3t^2}{3+2t} = 3 \Longrightarrow t^2 - 2t - 3 = 0 \\ &\Longrightarrow t = 3, t = -1 \end{aligned}$$

   Inserting the values of t in the equations for x, y we get the two points $(18, 15)$ and $(-2, -13)$.

2. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at the point $(1, 3)$ on the curve $x = e^{-t},\ y = 3\cos t$.

   Answer:

$$\begin{aligned} \frac{dy}{dx} &= \frac{-3\sin t}{-e^{-t}}, \\ \frac{d^2y}{dx^2} &= \frac{\frac{d}{dt}\frac{dy}{dx}}{\frac{dx}{dt}} = \frac{-e^{-t}(-3\cos t) + e^{-t}3\sin t}{-e^{-3t}} \\ &= \frac{3\cos t + 3\sin t}{-e^{-2t}} \end{aligned}$$

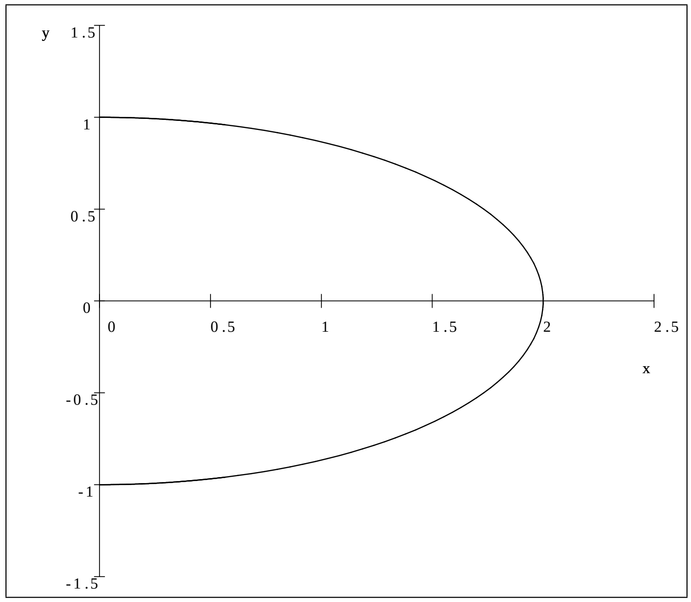
   At the point $(1, 3)$, $t = 0$ Therefore $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} = -3$.

3. Eleminate t and sketch the resulting curve for $x = 2\cos t, y = \sin t,\ -\frac{\pi}{2} < t < \frac{\pi}{2}$. Indicate with an arrow the direction in which the curve is traced as t increases.

Answer:
Eleminating t results in the equation

$$\frac{x^2}{4} + y^2 = 1.$$

For the given range of t, we only have the right half of the ellipse.



The curve is traced counterclockwise from $y = -1$ to $y = 1$.