1. Find and sketch the domain of the function $f(x,y) = \frac{\sqrt{x-y^2}}{1-x^2}$.

Solution:

In the domain of f, we must have $x - y^2 \ge 0$ and $x \ne \pm 1$. This gives all points inside or on the parabola $x = y^2$ but not on the line x = 1.



2. Find $\lim_{(x,y)\to(0,0)} \frac{6xy^3}{2x^4 + y^4}$ or show that it does not exist. Solution:

Approach through the line x = 0 to get

$$\lim_{y \to 0} \frac{0}{y^4} = 0.$$

Approach through the line y = x to get

$$\lim_{x \to 0} \frac{6x^4}{2x^4 + x^4} = 2.$$

Since the two limits are different, $\lim_{(x,y)\to(0,0)} \frac{6xy^3}{2x^4 + y^4}$ does not exist.

3. Find the first partial derivatives of the function $u = te^{w/t}$. Solution:

$$u_t = e^{w/t} - \frac{w}{t}e^{w/t}.$$
$$u_w = e^{w/t}.$$