

1. (a) (**3 points**) Find the center and radius of the sphere  $S : x^2 + y^2 + z^2 + 4x - 8y - 2z + 5 = 0$ .
- (b) (**2 points**) Find points  $A, B$  on the sphere  $S$  such that  $AB$  is a diagonal of the sphere.
- (c) (**2 points**) Find the distance from the origin to the sphere  $S$ .
2. (a) (**2 points**) Find the unit vector  $\mathbf{u}$  with direction angles  $\frac{1}{3}\pi, \frac{1}{4}\pi, \gamma$ , where  $\gamma$  is an obtuse angle.
- (b) (**2 points**) Find the angle between the vectors the vectors  $3\mathbf{i} + \mathbf{j} - 2\mathbf{k}$  and  $\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$  to the nearest hundredth radians.
- (c) (**2 points**) Find the vector projection of  $\mathbf{a} = 3\mathbf{i} + \mathbf{j} - 2\mathbf{k}$  onto the vector  $\mathbf{b} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$ .
- (d) (**2 points**) Show that if  $\mathbf{a} \cdot \mathbf{b} = \mathbf{a} \cdot \mathbf{c}$ , then  $\mathbf{b}$  and  $\mathbf{c}$  have the same vector projection onto  $\mathbf{a}$ .
3. (a) (**2 points**) Find the volume of the parallelepiped with edges along the vectors  $2\mathbf{i} + \mathbf{j}, 3\mathbf{i} - 2\mathbf{k}, 3\mathbf{j} + 2\mathbf{k}$ .
- (b) (**3 points**) Find the distance between the point  $P(1, 0, -1)$  and line through the points  $A(1, 2, 1), B(2, 2, -2)$ .
4. (a) (**3 points**) Determine whether  $l_1 : x = 3 + t, y = 1 - t, z = 5 + 2t$  and  $l_2 : x = 1, y = 4 - t, z = 9 - 2t$  intersect. If so, find their point of intersection.
- (b) (**2 points**) Find parametric equations of the line through  $P(1, 4, -3)$  and perpendicular to the  $yz$ -plane.
5. (a) (**3 points**) Find the equation of the plane through the point  $P(2, 0, 1)$  and contains the line  $l : x = 1 - 2t, y = 1 + 4t, z = 2 + t$ .
- (b) (**2 points**) Find the distance from the point  $P(2, -1, 3)$  to the plane  $2x + 4y - z + 1 = 0$ .