KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS
DEPARTMENT OF MATHEMATICS AND STATISTICS
MATH 640
Exam # 1

1. Let $F : V \to \overline{\mathbb{R}}$. Show that the set of continuous affine minorants of F is convex.
2. Let χ_A be the indicator function of the set A. Show that $\overline{\chi}_A = \chi_{\overline{A}}$ and $\chi_A^{**} = \chi_{\overline{co}A}$.
3. Let $F : V \to \overline{\mathbb{R}}$ and G be the Γ-regularization of F. Show that if $F(u) = \infty$ then $G(u) = \infty$ and if $F(u) = -\infty$ then $G \equiv -\infty$.
4. Show that if $F : V \to \overline{\mathbb{R}}$ is convex and G is a continuous affine function then $F - G$ is convex.
5. Show that if $F, G \in \Gamma(V)$, then $F + G \in \Gamma(V)$.
6. Show that if $F : V \to \overline{\mathbb{R}}$ is convex and $u, v \in V$ then the expression $\frac{F(u+\lambda v)-F(u)}{\lambda}$ is increasing in λ on $(0, \infty)$.