1. For the differential equation $y'' + 4y' + 4y = 4\cos 2x + 3\sin 2x - 8$, (i) find the solutions of the homogeneous equation and (ii) find an annihilator of lowest order for the right hand side.

   Solution:

   (i) The auxiliary equation $m^2 + 4m + 4 = 0$ has the repeated $m = -2$. The homogenous solutions then are: $y_1 = e^{2x}$ and $y_2 = xe^{2x}$.

   (ii) The operator $(D^2 + 4)$ kills both $\cos 2x$ and $\sin 2x$ and the operator D kills 8. Therefore, the required annihilator is $D\,(D^2 + 4)$.

2. Two roots of a cubic auxiliary equation with real coefficients are $m_1 = \frac{1}{2}$ and $m_2 = 1 + 3i$. What is the corresponding linear homogeneous differential equation?

   Solution:

   The auxiliary equation can be written as

   $$
   \begin{aligned}
   \left(m - \frac{1}{2}\right)(m - (1 + 3i))(m - (1 - 3i)) &= 0 \\
   \left(m - \frac{1}{2}\right)(m^2 - 2m + 10) &= 0 \\
   (2m - 1)(m^2 - 2m + 10) &= 0 \\
   2m^3 - 5m^2 + 22m - 10 &= 0,
   \end{aligned}
   $$

   from which we get the differential equation

   $$
   2y''' - 5y'' + 22y' - 10y = 0.
   $$

   Remark: observe that $(m - (\alpha + \beta i))(m - (\alpha - \beta i)) = m^2 - 2\alpha m + (\alpha^2 + \beta^2)$.