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MATH 301/Term 062/Hw#4(9.8)/

6. Let $G(x, y, z) = 4xyz$ be a real-valued function and let C be the curve defined by

$$\begin{cases} x = \frac{1}{3}t^3, \\ y = t^2, & t \in [0, 1]. \\ z = 2t, \end{cases}$$

It is clear that G is defined and differentiable everywhere. Then we have

$$\int_C G(x, y, z)dx = \int_0^1 \frac{4}{3}t^3t^2(2t)3\frac{1}{3}t^2dt = \int_0^1 \frac{8}{3}t^8dt = \Big[\frac{8}{3}\frac{t^9}{9}\Big]_0^1 = \frac{8}{27}.$$

$$\int_C G(x, y, z)dy = \int_0^1 \frac{4}{3}t^3t^2(2t)(2t)dt = \int_0^1 \frac{16}{3}t^7dt = \Big[\frac{16}{3}\frac{t^8}{8}\Big]_0^1 = \frac{2}{3}.$$

$$\int_C G(x, y, z)dz = \int_0^1 \frac{4}{3}t^3t^2(2t)(2)dt = \int_0^1 \frac{16}{3}t^6dt = \Big[\frac{16}{3}\frac{t^7}{7}\Big]_0^1 = \frac{16}{21}.$$

$$\begin{aligned}
\int_C G(x, y, z)ds &= \int_0^1 \frac{4}{3}t^3t^2(2t)\sqrt{(t^2)^2 + (2t)^2 + 2^2}dt \\
&= \int_0^1 \frac{8}{3}t^6\sqrt{t^4 + 4t^2 + 4}dt = \int_0^1 \frac{8}{3}t^6\sqrt{(t^2 + 2)^2}dt \\
&= \int_0^1 \frac{8}{3}t^6(t^2 + 2)dt = \int_0^1 \frac{8}{3}\Big(t^8 + 2t^6\Big)dt \\
&= \Big[\frac{8}{3}\Big(\frac{t^9}{9} + \frac{2t^7}{7}\Big)\Big]_0^1 = \frac{8}{3}\Big(\frac{1}{9} + \frac{2}{7}\Big) = \frac{200}{189}.
\end{aligned}$$

□

9. We would like to evaluate the line integral $\int_C (2x+y)dx + xydy$ along the piecewise smooth curve $C = C_1 \cup C_2$, where C_1 is the horizontal line segment joining the points $(-1,2)$ and $(2,2)$, and where C_2 is the vertical line segment joining the points $(2,2)$ and $(2,5)$. C_1 and C_2 have the parameterizations

$$C_1 : \begin{cases} x = t, \\ y = 2, \quad t \in [-1,2] \end{cases} \qquad \text{and} \qquad C_2 : \begin{cases} x = 2, \\ y = t, \quad t \in [2,5]. \end{cases}$$

Then we have

$$\int_C (2x+y)dx + xydy = \int_{C_1} (2x+y)dx + xydy + \int_{C_2} (2x+y)dx + xydy. \quad (1)$$

$$\begin{aligned} \int_{C_1} (2x+y)dx + xydy &= \int_{C_1} (2x+y)dx + \int_{C_1} xydy \\ &= \int_{-1}^{2} (2t+2)dt + \int_{-1}^{2} (2t)(0)dt \\ &= [t^2+2t]_{-1}^{2} = [(4+4)-(1-2)] = 9. \qquad (2) \end{aligned}$$

$$\begin{aligned} \int_{C_2} (2x+y)dx + xydy &= \int_{C_2} (2x+y)dx + \int_{C_2} xydy \\ &= \int_{2}^{5} (4+t)(0)dt + \int_{2}^{5} (2t)dt \\ &= [t^2+]_2^5 = [4-25] = -21. \qquad (3) \end{aligned}$$

Using (1), (2) and (3), we get

$$\int_C (2x+y)dx + xydy = 9 - 21 = -12.$$

□

16. We would like to evaluate the line integral $\int_C -y^2dx + xydy$ along the smooth curve Cdefined by

$$C : \begin{cases} x = 2t, \\ y = t^3, \quad t \in [0,2]. \end{cases}$$

We have

$$\int_C -y^2dx + xydy = \int_C -y^2dx + \int_C xydy. \tag{1}$$

$$\int_C -y^2dx = \int_0^2 -t^6(2)dt = \Big[-\frac{2t^7}{7}\Big]_0^2 = -\frac{256}{7}. \tag{2}$$

$$\int_C xydy = \int_0^2 (2t)t^3(3t^2)dt = \int_0^2 6t^6dt = \Big[\frac{6t^7}{7}\Big]_0^2 = \frac{768}{7}. \tag{3}$$

Using (1), (2) and (3), we get

$$\int_C -y^2dx + xydy = \frac{768}{7} - \frac{256}{7} = \frac{512}{7}.$$

□

30. Let $\mathbf{F}(x,y,z) = e^x\mathbf{i} + xe^{xy}\mathbf{j} + xye^{xyz}\mathbf{k}$ and $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$, where $0 \le t \le 1$, be two vector functions. We would like to evaluate the line integral $\displaystyle\int_C \mathbf{F}.d\mathbf{r}$.

The vector function $\mathbf{r}(t)$ gives the parametric equations of the curve C as follows

$$C : \begin{cases} x = t, \\ y = t^2, \quad t \in [0,1]. \\ z = t^3. \end{cases}$$

Then we have

$$\begin{aligned}
\int_C \mathbf{F}.d\mathbf{r} &= \int_C (e^x\mathbf{i} + xe^{xy}\mathbf{j} + xye^{xyz}\mathbf{k}).(dx\mathbf{i} + dy\mathbf{j} + dz\mathbf{k}) \\
&= \int_C e^xdx + xe^{xy}dy + xye^{xyz}dz \\
&= \int_0^1 e^tdt + \int_0^1 2t^2e^{t^3}dt + \int_0^1 3t^5e^{t^6}dt \\
&= \Big[e^t + \frac{2}{3}e^{t^3} + \frac{3}{6}e^{t^6}\Big]_0^1 \\
&= e + \frac{2}{3}e + \frac{1}{2}e - 1 - \frac{2}{3} - \frac{1}{2} = \frac{13}{6}(e-1).
\end{aligned}$$