Department of Mathematical Sciences
KFUPM
Term 031

MATH 301-01/ Final Exam/ Duration=3hours

1. Show that $\int_C \left(-y\sin(xy) + \frac{x}{x^2+y^2+1}\right)dx + \left(-x\sin(xy) + \frac{y}{x^2+y^2+1}\right)dy$ is independent of path and evaluate the line integral from $A=(0,0)$ to $B=(1,\pi)$.

2. Evaluate $\int\int_S \overrightarrow{F}.\overrightarrow{n}\,dS$, where $\overrightarrow{F} = x\,\overrightarrow{i} + y\,\overrightarrow{j} + z\,\overrightarrow{k}$, S is the exterior surface of the region D that is above the xy-plane and below the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$, and $\overrightarrow{n}$ is the outward unit normal vector to S. (Use the change of variables $x = ra\cos(\theta)$, $y = rb\sin(\theta)$, $0 \le r \le 1$, $0 \le \theta \le 2\pi$).)
Use the divergence theorem to show that the volume of the region bounded by the ellipsoid is $\frac{4\pi}{3}abc$.

3. Solve the initial-value problem:

$$y''(t) - 2y(t) = \mathcal{U}_\pi(t), \qquad y(0)=0,\ y'(0)=1.$$

4. Find $\mathcal{L}\left\{e^{2t}\sin(t)\mathcal{U}_\pi(t)\right\}$ and $\mathcal{L}^{-1}\left\{\frac{s+1}{s^2+s+1}\right\}$.

5. Expand $f(x)=x^2$ in a half-range cosine series in $[0,\pi]$. Show that $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k^2} = \frac{\pi^2}{12}$.

6. Find the eigenvalues and eigenfunctions of the boundary-value problem:

$y'' + 2ay' + \lambda y = 0, \quad y'(0)=0,\ y'(L)=0,$ where a and L are given positive numbers.

7. Use the method of separation of variables to solve the following boundary-value problem

$$\begin{cases} \frac{\partial^2 u}{\partial x^2} + 4\frac{\partial^2 u}{\partial y^2} = 0 & \text{for } 0<x<3,\ 0<y<4, \\ u(0,y)=0, \quad \frac{\partial u}{\partial x}(3,y)=0 & \text{for } 0<y<4, \\ u(x,0)=x, \quad u(x,4)=0 & \text{for } 0<x<3. \end{cases}$$

8. Use one of the Fourier integral transforms to solve the following boundary-value problem

$$\begin{cases} \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2} & \text{for } t>0,\ 0<x<1, \\ u(x,0)=0, \quad \lim_{t\to+\infty} u(x,t)=0 & \text{for } 0<x<1, \\ \frac{\partial u}{\partial x}(0,t)=e^{-t}, \quad \frac{\partial u}{\partial x}(1,t)=e^{-t}, & \text{for } t>0. \end{cases}$$

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