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$\dot{x} = f(t, x)$

Automous systems

Linear ($\dot{x} = Ax$)

Eigenvalues analysis for the stability of the solution $x = 0$

Non − linear($\dot{x} = f(x)$)

The first integral $F(x, y)$ tells us about the degenerate points, existence of saddles and centers (periodic solutions)

If $f(x) = Ax + g(x) \ni \lim_{\|x\|\to 0}(\|g(x)\|/\|x\|) = 0$ then linearization characterizes saddles and attractors

We use Bendixson's criterion for non existence of periodic solutions and attractors

By Poincare' Bendixson's theorem, we check the existence of periodic solutions

Check out for equations of ***Leinard and Van der Pol***

Stability

For solution $x = 0$; $\lim_{t\to\infty}(\alpha(t)) = 0$ For every orbit $\alpha(t)$ which start in its neighborhood.

For periodic solution we check out for Lyapunov stability, orbital stability and asymptotic stability. If we have the periodic solution $\varphi(t)$, then we can apply theorem 7.4

Non − Autonomous systems

Linear

If $\dot{x} = Ax + B(t)x$, then we apply one of the theorems 6.2, 6.3 or 6.4 if *applicable*, to analyze Lyapunov, asymptotic stability or instability respectivel. for the solution $x = 0$.

If $\dot{x} = A(t)x$ with $A(t)$ periodic, then we use exponents to analyze the stability of the periodic and non trivial solutions.

Non − linear

If $\dot{x} = Ax + B(t)x + f(t, x)$, then we apply the **Poincare' Lyapunov** if *applicable* to analyze the stability of the trivial solution

If $\dot{x} = A(t)x + f(t, x)$, with $A(t)$ T −periodic, then we apply theorem 7.2 if *applicable* to analyze the stability of the trivial solution.

If $f(t, x)$ is T −periodic and $A(t)$ continuous in t, with $\dot{y} = A(t)y$ having no T −periodic except for $\dot{y} = y = 0$, then $\dot{x}$ has at least one periodic solution. Theorem 4.9.

If we have the Lyapunov function $V(t, x)$, then we can use the direct method to analyze the stability of the equilibrium solution. For Hamiltonian system, we apply the theory of stability analysis of the solution for such systems.