King Fahd University of Petroleum & Minerals
Department of Mathematics and Statistics

CODE 003 | **Math 101** | CODE 003

Exam 2

103

August 2, 2011

Net Time Allowed: 120 minutes

Name: _____

ID: _____ Sec: _____.

Check that this exam has 20 questions.

Important Instructions:

1. All types of calculators, pagers or mobile phones are NOT allowed during the examination.
2. Use HB 2.5 pencils only.
3. Use a good eraser. DO NOT use the erasers attached to the pencil.
4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
5. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. If $y = x^x + 2^x + x^2$, then $y' =$

(a) $xx^{x-1} + x2^{x-1} + 2x$

(b) $x^x(\ln x + 1) + x2^{x-1} + 2x$

(c) $xx^{x-1} + 2^x \ln 2 + 2x$

(d) $x^x(\ln x + 1) + 2^x \ln 2 + 2x$

(e) $x^x \ln x + 2^x \ln 2 + 2x$

2. If $f(x) = \dfrac{h(x) + x}{x + 1}$, $f'(1) = \dfrac{1}{2}$, and $h(1) = 1$, then $h'(1) =$

(a) 0

(b) 1

(c) $\dfrac{1}{2}$

(d) -1

(e) $\dfrac{3}{2}$

3. If $(0, \beta)$ is a point on the tangent line to the graph of $y = -\pi + 4\tan^{-1}\left(\frac{2}{x}\right)$ at $x = 2$, then $\beta =$

(a) 5

(b) 6

(c) 4

(d) 3

(e) 2

4. The slope of the normal line to the curve $e^y \sin x = e^x \sin y$ at $\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$ is equal to

(a) $-e^{-\frac{\pi}{4}}$

(b) 2

(c) $e^{\frac{\pi}{4}}$

(d) -1

(e) 0

5. The radius of a circular disk is given as 5 cm with a maximum error in measurement of 0.1 cm. Using differentials, the maximum error in the calculated area of the disk is

(a) 10π cm^2

(b) π cm^2

(c) 0.5π cm^2

(d) 0.2π cm^2

(e) 0.1π cm^2

6. If $f(x) = (x-1)^{\frac{1}{3}}$, then the equation of the vertical tangent to the graph of f is

(a) None of these

(b) $x = -1$

(c) $x = 1$

(d) $x = \frac{1}{3}$

(e) $x = -\frac{1}{3}$

7. If the tangent line to the parabola $y = x^2 - 1$ is perpendicular to the tangent line of the parabola $y = ax^2 + 1$ at each intersection point, then $a =$

(a) $-\frac{1}{7}$

(b) $-\frac{1}{9}$

(c) $\frac{1}{9}$

(d) $\frac{1}{7}$

(e) $-\frac{1}{8}$

8. Consider the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$. The shaded area formed by the two axes and the tangent line to the ellipse at (a, b), has area

(a) $\frac{ab(1-a)^2}{a-2}$

(b) ab

(c) $\frac{a^2(1+b^2)}{b^2(1+a^2)}$

(d) $\frac{a^2(1+b^2)}{2b^2(1+a^2)}$

(e) $2ab$

9. If $y = e^z$, $z = \ln u^2$ and $u^2 + 1 = \tan x$ then $\left.\frac{dy}{dx}\right|_{x=\pi/4}$ is

(a) $\frac{1}{\sqrt{2}}$

(b) None of these

(c) $\frac{1}{2}$

(d) 2

(e) $\sqrt{2}$

10. The curve $y = x^3 + x^2 - x$ has two horizontal tangents at $x = a$ and $x = b$. Then $a + b =$

(a) $-\frac{2}{3}$

(b) 0

(c) $\frac{5}{3}$

(d) $\frac{1}{3}$

(e) $\frac{1}{2}$

11. $\lim_{x \to 0} \dfrac{x - x\cos 3x}{\sin^2 2x} =$

(a) 3

(b) $\frac{1}{4}$

(c) $\frac{3}{2}$

(d) $\frac{3}{4}$

(e) 0

12. If $y = \sec^2 x$, then $y'' =$

(a) None of these

(b) $-2y^2 + 4y$

(c) $-2y^2 - 4y$

(d) $6y^2 - 4y$

(e) $6y^2 + 4y$

13. If linear approximation is used to approximate $\cos(59°)$, we get $\cos(59°) \approx a + b\left(\frac{\pi}{180}\right)$, then $2a + \frac{2}{\sqrt{3}}b$ is equal to

(a) 5

(b) 2

(c) 3

(d) 6

(e) 4

14. A particle moves according to the law of motion $s(t) = \ln(1 + t^2)$ for t in $[0, \sqrt{2})$, where t is measured in seconds and s in meters. The particle is speeding up when

(a) $0 < t < \sqrt{2}$

(b) $1 < t < \sqrt{2}$

(c) $0 < t < 1$

(d) $0 \leq t < \sqrt{2}$

(e) None of these

15. If $y = \dfrac{(x-1)^4(2x-1)^5}{(3x-1)^3(10x+1)^7}$, then $\left.\dfrac{dy}{dx}\right|_{x=0} =$

(a) -47

(b) -75

(c) 75

(d) 0

(e) 47

16. The velocity of a particle in motion along a line is $v(t) = \ln|2-t^2|$ for t in $[0, \sqrt{2})$. Find the acceleration when the object is at rest.

(a) -1

(b) -2

(c) 2

(d) 1

(e) None of these

17. $\lim_{x\to 0}(1+2x)^{\frac{3}{x}} =$

(a) $e^{\frac{2}{3}}$

(b) $e^{\frac{3}{2}}$

(c) e^3

(d) 1

(e) e^6

18. If $f(x) = \ln(1-x)$, then $f^{(2011)}(0)$ is equal to

(a) $(2011)!$

(b) $\ln(2011)$

(c) $(2012)!$

(d) $-(2010)!$

(e) $-2009!$

19. A right circular cone has a base with radius r, and height h. If the radius is expanding at a rate of 2 mm/hr, while the height is contracting/shrinking at the same rate, then the volume will stay constant if

[Hint: $V = \frac{1}{3}\pi r^2 h$]

(a) $r^2 = h$

(b) $2h + r = 0$

(c) $r = h$

(d) $h = 2r$

(e) $r = 2h$

20. When a stone is dropped into a pool, a circular wave moves out from the point of impact at the rate of $3a$ meter per second ($a > 0$ a real constant). How fast is the area enclosed by the wave increasing when the radius of the wave is a meter?

(a) $3\pi a$ m$^2/s$

(b) $6\pi a$ m$^2/s$

(c) $6\pi a^2$ m$^2/s$

(d) $3\pi a^2$ m$^2/s$

(e) $2\pi a^2$ m$^2/s$