8.1 Integration by Parts

<u>Formula:</u> $\int f(x)\, g'(x)\, dx = f(x)\, g(x) - \int g(x)\, f'(x)\, dx$

or $\int u\, dv = uv - \int v\, du$

<u>Ex1</u> Find $\int x \sin x \, dx$

$u = x \qquad dv = \sin x \, dx$

$du = dx \qquad v = -\cos x$

$$\int x \sin x \, dx = x(-\cos x) - \int (-\cos x)\, dx$$
$$= -x\cos x + \int \cos x \, dx$$
$$= -x\cos x + \sin x + C.$$

<u>Remark:</u> Try to choose u and dv in such a way that gives an easier integral than the original one.

<u>Ex_2</u> Evaluate $\int \ln x \, dx$

$u = \ln x \qquad dv = dx$

$du = \frac{1}{x} dx \qquad v = x$

$$\int \ln x \, dx = x\ln x - \int x \frac{dx}{x}$$

$$= x\ln x - \int dx = x\ln x - x + C$$

EX3 Find $\int t^2 e^t \, dt$

$u = t^2 \qquad dv = e^t \, dt$

$du = 2t \, dt \qquad v = e^t$

$$\int t^2 e^t \, dt = t^2 e^t - \int 2t e^t \, dt$$

$$= t^2 e^t - 2\int t e^t \, dt$$

$$= t^2 e^t - 2\left[t e^t - \int e^t \, dt \right]$$

$$= t^2 e^t - 2te^t + 2e^t + C$$

EX4 Evaluate $\int e^x \sin x \, dx$

let $\quad u = e^x \qquad dv = \sin x \, dx$

$du = e^x \, dx \qquad V = -\cos x$

$$I = -\cos x \, e^x - \int -\cos x \, e^x \, dx$$

$$= -\cos x \, e^x + \int \cos x \, e^x \, dx$$

$u = e^x \qquad dv = \cos x \, dx$

$du = e^x \, dx \qquad v = \sin x$

$$I = -\cos x\, e^x + e^x \sin x - \int e^x \sin x \, dx$$

$$\Rightarrow 2I = e^x \left[\sin x - \cos x\right] + C$$

$$I = \frac{e^x}{2}\left[\sin x - \cos x\right] + C_1$$

<u>Remark</u>: $$\int_a^b f(x)\, g'(x)\, dx = f(x)\, g(x) \Big|_a^b - \int_a^b g(x)\, f'(x)\, dx$$

<u>Ex5</u> Calculate $\int_0^1 \tan^{-1} x \, dx$

$$u = \tan^{-1} x \qquad dv = dx$$

<u>Ex6</u> Prove the reduction formula

$$\int \sin^n x \, dx = -\frac{1}{n} \cos x \sin^{n-1} x + \frac{n-1}{n} \int \sin^{n-2} x \, dx$$

let $u = \sin^{n-1} x \qquad dv = \sin x \, dx$

$\boxed{du = (n-1) \sin^{n-2} x \cos x \, dx} \qquad \boxed{v = -\cos x}$

then use $\cos^2 x = 1 - \sin^2 x$.

Ex7 $\int \sin^{-1} x \, dx$ let $u = \sin^{-1} x$ $dv = dx$

$= x \sin^{-1} x - \int x \dfrac{1}{\sqrt{1-x^2}} \, dx$ ...

Ex8 $\int_1^4 \sqrt{t} \ln t \, dt$

$u = \ln t$ $dv = \sqrt{t} \, dt$.

Ex9 $\int \cos(\ln x) \, dx$

$u = \cos(\ln x)$ $dv = dx$

$du = \frac{-1}{x} \sin(\ln x) \, dx$ $v = x$

$I = \int \cos(\ln x) \, dx = x \cos(\ln x) + \int \sin(\ln x) \, dx$

$I = x \cos(\ln x) + x \sin(\ln x) - \int \cancel{x} \cdot \frac{1}{\cancel{x}} \cos(\ln x) \, dx$

$\Rightarrow 2I = x \cos(\ln x) + x \sin(\ln x)$

$\Rightarrow I = \frac{1}{2} \left[x \cos(\ln x) + x \sin(\ln x) \right] + C$.

First make a substitution and then use integration by parts to evaluate

a) $\int_1^4 e^{\sqrt{x}}\, dx$

Soln: let $w = \sqrt{x} \Rightarrow dw = \frac{1}{2\sqrt{x}}\, dx$

$$\Rightarrow dx = 2\sqrt{x}\, dw = 2w\, dw$$

$$\therefore I = \int_1^2 2w\, e^w\, dw$$

$$= 2\left[w e^w \Big|_1^2 - \int_1^2 e^w\, dw \right]$$

$$= 2\left[(2e^2 - e) - (e^2 - e) \right]$$

$$= 2\left[e^2 \right] = 2e^2.$$

b) $\int x^5 e^{x^2}\, dx$ let $w = x^2 \Rightarrow dw = 2x\, dx$

$$I = \int (x^2)^2 . x . e^{x^2}\, dx$$

w^2	e^w
$2w$	e^w
2	e^w
0	e^w

$$= \frac{1}{2}\int w^2 e^w\, dw$$

$$= \frac{1}{2}\left[w^2 e^w - 2w e^w + 2e^w \right] = \frac{1}{2}\left[x^4 e^{x^2} - 2x^2 e^{x^2} + 2e^{x^2} \right] + C$$

Suppose that $f(1) = 2$, $f(4) = 7$, $f'(1) = 5$, $f'(4) = 3$ and f'' is continuous. Find the value of $\int_1^4 x f''(x)\, dx$

Soln:

$$\int_1^4 x f''(x)\, dx = x f'(x) \Big|_1^4 - \int_1^4 f'(x)\, dx$$

$$= \left[4 f'(4) - f'(1)\right] - \left[f(4) - f(1)\right]$$

$$= (12 - 5) - (7 - 2)$$

$$= 7 - 5 = 2 .$$

8.2 Trigonometric Integrals

Recall:

(1) $\sin^2 x + \cos^2 x = 1$

(2) $1 + \tan^2 x = \sec^2 x$

(3) $\cos^2 x = \dfrac{1 + \cos 2x}{2}$

(4) $\sin^2 x = \dfrac{1 - \cos 2x}{2}$

(5) $\sin 2x = 2 \sin x \cos x$

EX1 Evaluate $\int \cos^3 x \, dx$ (Power of cosine is odd)

soln: $\int \cos^3 x \, dx = \int \cos^2 x \cos x \, dx$

$$= \int (1 - \sin^2 x) \cos x \, dx$$

$$= \int \cos x \, dx - \int \sin^2 x \cos x \, dx$$

let $u = \sin x$
$du = \cos x \, dx$

$$= \cos x - \frac{1}{3} \sin^3 x + C.$$

EX2 Find $\int \sin^5 x \cos^2 x \, dx$ (Power of sine is odd)

Soln: $\int \sin^4 x \sin x \cos^2 x \, dx$

$$= \int (1-\cos^2 x)^2 \cos^2 x \ \sin x \ dx$$

let $u = \cos x \qquad du = -\sin x \ dx$

$$= \int (1-u^2)^2 u^2 (-du)$$

$$= -\int (1-2u^2+u^4) u^2 \ du \ = \int u^2 - 2u^4 + u^6 \ du$$

$$= -\left(\frac{u^3}{3} - 2\frac{u^5}{5} + \frac{u^7}{7} \right) + C \ = -\left(\frac{\cos^3 x}{3} - 2\frac{\cos^5 x}{5} + \frac{\cos^7 x}{7} \right) + C$$

EX3 Evaluate $\displaystyle\int_0^{\pi} \sin^2 x \ dx$ (Power of sine is even)

$$= \int_0^{\pi} \frac{1-\cos 2x}{2} \ dx$$

$$= \frac{1}{2} \int_0^{\pi} (1-\cos 2x) dx = \frac{1}{2} \left[x - \tfrac{1}{2}\sin 2x \Big|_0^{\pi} \right]$$

$$= \frac{1}{2} \left[\left(\pi - \tfrac{1}{2}\sin 2\pi\right) - \left(0 - \tfrac{1}{2}\sin 0\right) \right]$$

$$= \frac{\pi}{2}.$$

EX4 Find $\int \sin^4 x \ dx$ (power of sine is even).

sln:

$$\int \sin^4 x\, dx = \int (\sin^2 x)^2\, dx$$

$$= \int \left(\frac{1-\cos 2x}{2}\right)^2 dx$$

$$= \frac{1}{4}\int (1-\cos 2x)^2\, dx$$

$$= \frac{1}{4}\int 1 - 2\cos 2x + \cos^2 2x\ \ dx$$

$$= \frac{1}{4}\int 1 - 2\cos 2x + \frac{1+\cos 4x}{2}\ \ dx$$

$$= \frac{1}{4}\int\left(\frac{3}{2} - 2\cos 2x + \frac{1}{2}\cos 4x\right) dx$$

$$= \frac{1}{4}\left(\frac{3}{2}x - \sin 2x + \frac{1}{8}\sin 4x\right) + C$$

Remark: Read page 462 : strategy for evaluating $\int \sin^m x \cos^n x\, dx$.

EX 5 Evaluate $\int \tan^6 x \sec^4 x\ dx$ (Power of sec is even)

$$= \int \tan^6 x\ \sec^2 x\ \sec^2 x\, dx$$

$$= \int \tan^6 x\ (1+\tan^2 x)\ \sec^2 x\, dx$$

let $u = \tan x$
$du = \sec^2 x\, dx$

$$= \int u^6 (1+u^2)\, du = \frac{u^7}{7} + \frac{u^9}{9} + C$$

$$= \frac{1}{7}\tan^7 x + \frac{1}{9}\tan^9 x + C$$

EX6 Find $\int \tan^5\theta \sec^7\theta \, d\theta$ (Power of tangent is odd)

$$= \int \tan^4\theta \sec^6\theta \tan\theta \sec\theta \, d\theta$$

$$= \int (\sec^2\theta - 1)^2 \sec^6\theta \tan\theta \sec\theta \, d\theta$$

let $u = \sec\theta \Rightarrow du = \tan\theta \sec\theta \, d\theta$

$$= \int (u^2-1)^2 u^6 \, du = \int u^{10} - 2u^8 + u^6 \, du$$

$$= \frac{u^{11}}{11} + \frac{2}{9}u^9 + \frac{u^7}{7} + C = \frac{1}{11}\sec^{11}\theta - \frac{2}{9}\sec^9\theta + \frac{1}{7}\sec^7\theta + C.$$

__Formulas__

$$\int \tan x \, dx = \ln|\sec x| + C$$

$$\int \sec x \, dx = \ln|\sec x + \tan x| + C.$$

EX7 Find $\int \tan^3 x \, dx$

__soln:__ $\int \tan^3 x \, dx = \int \tan x \tan^2 x \, dx$

$$= \int \tan x \, (\sec^2 x - 1) \, dx$$

$$= \int \tan x \sec^2 x \, dx - \int \tan x \, dx$$

$$= \frac{\tan^2 x}{2} - \ln|\sec x| + C.$$

<u>EX8</u> Find $\int \sec^3 x \, dx$

$$= \int \sec x \sec^2 x \, dx$$

Let $u = \sec x$ $\quad dv = \sec^2 x \, dx$

Final Answer: $\int \sec^3 x \, dx = \frac{1}{2}\left(\sec x \tan x + \ln|\sec x + \tan x|\right) + C.$

EFS

$$\int \csc^4 x \cot^6 x \, dx \qquad 1 + \cot^2 x = \csc^2 x$$

(power of csc. is even)

$$\int \csc^2 x \left(1 + \cot^2 x\right) \cot^6 x \, dx$$

let $u = \cot x$

$du = -\csc^2 x \, dx$

EFS

$$\int \cot^3 \alpha \csc^3 \alpha \, d\alpha \qquad \text{(power of cotangent is odd.)}$$

$$= \int \cot^2 \alpha \csc^2 \alpha \, (\cot \alpha \csc \alpha) \, d\alpha$$

$$= \int (\csc^2 \alpha - 1) \csc^2 \alpha \, (\cot \alpha \csc \alpha) \, d\alpha$$

let $u = \csc \alpha \Rightarrow du = -\cot \alpha \csc \alpha \, d\alpha$

$$= -\int (u^2 - 1)(u^2) \, du.$$

Remember:

(a) $\sin(A) \cos(B) = \frac{1}{2} \left[\sin(A-B) + \sin(A+B) \right]$

(b) $\sin(A) \sin(B) = \frac{1}{2} \left[\cos(A-B) - \cos(A+B) \right]$

(c) $\cos A \cos B = \frac{1}{2} \left[\cos(A-B) + \cos(A+B) \right]$

EX9 Evaluate $\int \sin(4x) \cos(5x) \, dx$

$$= \frac{1}{2} \int \left[\sin(4x - 5x) + \sin(4x + 5x) \right] dx$$

$$= \frac{1}{2} \int (\sin(-x) + \sin 9x) \, dx$$

$$= \frac{1}{2} \left(\cos x - \frac{1}{9} \cos 9x \right) + C$$

Q1 $\int \tan^3(2x)\sec^5(2x)\,dx$

let $w = 2x \Rightarrow dw = 2\,dx$

$$= \frac{1}{2}\int \tan^3 w \sec^5 w \, dw$$

$$= \frac{1}{2}\int \tan^2 w \sec^4 w \tan w \sec w \, dw \quad \ldots \qquad 1+\tan^2 w = \sec^2 w.$$

Q2 $\int \frac{1-\sin x}{\cos x}\,dx = \int (\sec x - \tan x)\,dx$

$$= \ln|\sec x + \tan x| - \ln|\sec x| + C$$

Q3 $\int \csc x \, dx = \int \csc x \, \frac{\csc x - \cot x}{\csc x - \cot x}\,dx$

let $u = \csc x - \cot x$.

Q6 $\int \tan^6(ay)\,dy$

Q4 $\int \frac{\cos x + \sin x}{\sin 2x}\,dx$

Q5 $\int t \sec^2(t^2)\tan^4(t^2)\,dt$

$$\int_{-\pi}^{\pi} \sin mx \cos nx \, dx = 0 \qquad (\text{odd}) \qquad (\text{Prove})$$

$$\int_{-\pi}^{\pi} \sin mx \sin nx \, dx = \begin{cases} 0 & \text{if } m \neq n \\ \pi & \text{if } m = n . \end{cases} \qquad (\text{Prove this})$$

$$2\int_{0}^{\pi} \sin mx \sin nx \, dx = 2\int_{0}^{\pi} \frac{\cos(m-n)x - \cos(m+n)x}{2} \, dx$$

$$= \int_{0}^{\pi} \cos(m-n)x - \cos(m+n)x \, dx$$

$$= \frac{1}{m-n} \sin(m-n)x \Big|_{0}^{\pi} - \frac{1}{m+n} \sin(m+n)x \Big|_{0}^{\pi}$$

$$= 0 \quad \text{if} \quad m \neq n$$

if $m = n$

$$\Rightarrow I = \int_{0}^{\pi} 1 - \cos(2n) \, dx = \pi - \frac{1}{2n} \sin 2n \Big|_{0}^{\pi}$$

$$= \pi .$$

END.

8.3 Trigonometric Substitution

Table of Trigonometric substitution

Expression	Substitution		Identity
$\sqrt{a^2-x^2}$	$x=a\sin\theta$	$-\frac{\pi}{2}\le\theta\le\frac{\pi}{2}$	$1-\sin^2\theta=\cos^2\theta$
$\sqrt{a^2+x^2}$	$x=a\tan\theta$	$-\frac{\pi}{2}<\theta<\frac{\pi}{2}$	$1+\tan^2\theta=\sec^2\theta$
$\sqrt{x^2-a^2}$	$x=a\sec\theta$	$0\le\theta<\frac{\pi}{2}$ or $\pi\le\theta<\frac{3\pi}{2}$	$\sec^2\theta-1=\tan^2\theta$

EX1 / 498 Evaluate $\displaystyle\int\frac{\sqrt{9-x^2}}{x^2}\,dx$

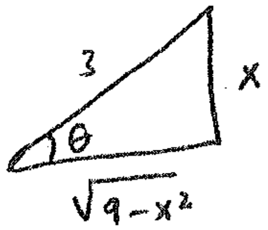
Soln: let $x=3\sin\theta$ $\quad -\frac{\pi}{2}\le\theta\le\frac{\pi}{2}$

$\Rightarrow dx=3\cos\theta\,d\theta$ and $\sqrt{9-x^2}=\sqrt{9-9\sin^2\theta}=3\cos\theta$

$$\int\frac{\sqrt{9-x^2}}{x^2}\,dx=\int\frac{3\cos\theta}{9\sin^2\theta}\,3\cos\theta\,d\theta$$

$$=\int\frac{\cos^2\theta}{\sin^2\theta}\,d\theta=\int\cot^2\theta\,d\theta$$

$$=\int(\csc^2\theta-1)\,d\theta=-\cot\theta-\theta+C$$

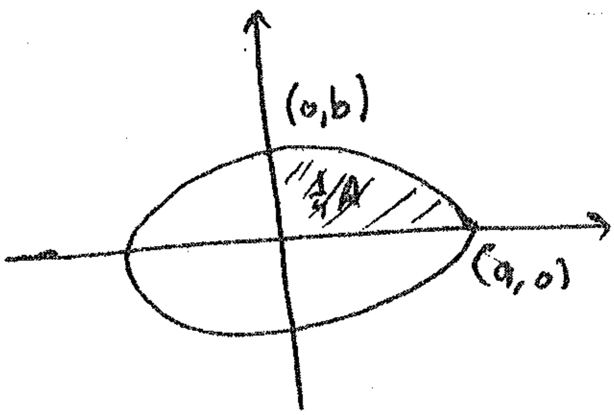


$$\Rightarrow \cot\theta = \frac{\sqrt{9-x^2}}{x} \qquad \text{and} \quad \theta = \sin^{-1}\frac{x}{3}$$

$$\Rightarrow \int \frac{\sqrt{9-x^2}}{x^2}\,dx = -\frac{\sqrt{9-x^2}}{x} - \sin^{-1}\left(\frac{x}{3}\right) + C.$$

<u>EX2</u> Find the Area enclosed by the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



Solving for y, we get

$$y = \pm \frac{b}{a}\sqrt{a^2-x^2}$$

$$\Rightarrow \frac{1}{4}A = \int_0^a \frac{b}{a}\sqrt{a^2-x^2}\,dx$$

Let $x = a\sin\theta \Rightarrow dx = a\cos\theta\,d\theta$

$$\frac{1}{4}A = \int_0^{\frac{\pi}{2}} \frac{b}{a}\sqrt{a^2-a^2\sin^2\theta}\;a\cos\theta\,d\theta$$

$$\Rightarrow \frac{1}{4}A = b\int_0^{\frac{\pi}{2}} a\cos^2\theta \, d\theta$$

$$= ab\int_0^{\frac{\pi}{2}} \frac{1+\cos 2\theta}{2}\, d\theta$$

$$\Rightarrow A = 2ab\left[\theta + \frac{1}{2}\sin 2\theta \Big|_0^{\frac{\pi}{2}}\right]$$

$$= 2ab\left[\frac{\pi}{2} + \frac{1}{2}\sin\pi - 0 - 0\right]$$

$$= 2ab\left(\frac{\pi}{2}\right) = \pi ab.$$

<u>EX3</u> Find $\int \frac{1}{x^2\sqrt{x^2+4}}\, dx$

<u>soln</u>: let $x = 2\tan\theta$, $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$.

$dx = 2\sec^2\theta \, d\theta$ and $\sqrt{x^2+4} = \sqrt{4(\tan^2\theta+1)}$

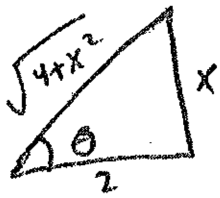
$= 2\sec\theta$

$$\Rightarrow \int \frac{dx}{x^2\sqrt{x^2+4}} = \int \frac{2\sec^2\theta \, d\theta}{4\tan^2\theta \, 2\sec\theta} = \frac{1}{4}\int \frac{\sec\theta}{\tan^2\theta}\, d\theta$$

$$= \frac{1}{4}\int \frac{\frac{1}{\cos\theta}}{\frac{\sin^2\theta}{\cos^2\theta}}\, d\theta = \frac{1}{4}\int \frac{\cos\theta}{\sin^2\theta}\, d\theta$$

let $u = \sin\theta \Rightarrow du = \cos\theta \, d\theta$

$$\Rightarrow \int \frac{dx}{x^2\sqrt{x^2+4}} = \frac{1}{4}\int u^{-2}\, du$$



$$= \frac{1}{4}\left(\frac{-1}{u}\right) + C$$

$$= -\frac{1}{4}\cdot \csc\theta + C$$

$$= -\frac{1}{4}\cdot \frac{\sqrt{x^2+4}}{\cancel{x}} + C$$

$$= -\frac{\sqrt{x^2+4}}{4x} + C$$

$\frac{\text{EX}_4}{490}$ Find $\int \frac{x}{\sqrt{x^2+4}}\, dx$

<u>soln</u>: let $u = x^2+4 \Rightarrow du = 2x\, dx$

$$\frac{1}{2}\int \frac{du}{u^{\frac{1}{2}}} = \sqrt{u} + C = \sqrt{x^2+4} + C$$

Remark: why Not $x = 2\tan\theta$?

EX5 Evaluate $\int \frac{dx}{\sqrt{x^2-a^2}}$ where $a>0$

soln: let $x = a\sec\theta$ where $0<\theta<\frac{\pi}{2}$ or $\pi<\theta<\frac{3\pi}{2}$

$dx = a\sec\theta\tan\theta\, d\theta$

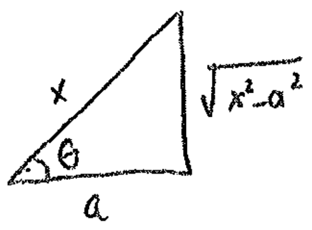
$$\sqrt{x^2-a^2} = \sqrt{a^2\sec^2\theta - a^2} = a\sqrt{\sec^2\theta - 1}$$

$$= a\,|\tan\theta|$$

$$= a\tan\theta$$

$$\Rightarrow \int \frac{dx}{\sqrt{x^2-a^2}} = \int \frac{a\sec\theta\tan\theta}{a\tan\theta}\, d\theta$$

$$= \int \sec\theta\, d\theta = \ln|\sec\theta + \tan\theta| + C$$



$\tan\theta = \frac{\sqrt{x^2-a^2}}{a}$, $\sec\theta = \frac{x}{a}$.

$$\Rightarrow \int \frac{dx}{\sqrt{x^2-a^2}}\, dx = \ln\left|\frac{x}{a} + \frac{\sqrt{x^2-a^2}}{a}\right| + C$$

$$= \ln\left|\frac{x+\sqrt{x^2-a^2}}{a}\right| + C$$

$$= \ln\left|x+\sqrt{x^2-a^2}\right| - \ln a + C$$

$$= \ln\left|x+\sqrt{x^2-a^2}\right| + C$$

EX6 Find $\displaystyle\int_0^{\frac{3\sqrt{3}}{2}} \frac{x^3}{(4x^2+9)^{3/2}}\,dx$

$$= \int_0^{\frac{3\sqrt{3}}{2}} \frac{x^3}{\left(\sqrt{4x^2+9}\right)^3}\,dx = \int_0^{\frac{3\sqrt{3}}{2}} \frac{x^3}{\left(\sqrt{(2x)^2+9}\right)^3}\,dx$$

let $u = 2x \Rightarrow du = 2\,dx$

$$\Rightarrow I = \int_0^{3\sqrt{3}} \frac{\frac{1}{8}u^3}{\left(\sqrt{u^2+9}\right)^3}\,\frac{du}{2}$$

$$I = \frac{1}{16}\int_0^{3\sqrt{3}} \frac{u^3}{(u^2+9)^{3/2}}\,du$$

$u = 3\tan\theta$ where θ in quadrant I or IV.

$du = 3\sec^2\theta\,d\theta$

$$\Rightarrow I = \frac{1}{16}\int_0^{\frac{\pi}{3}} \frac{27\tan^3\theta}{(9\tan^2\theta+9)^{3/2}}\cdot 3\sec^2\theta\,d\theta$$

$$= \frac{1}{16}\int_0^{\frac{\pi}{3}} \frac{27\tan^3\theta\cdot 3\sec^2\theta}{27\sec^3\theta}\,d\theta$$

$$= \frac{3}{16}\int_0^{\frac{\pi}{3}} \frac{\tan^3\theta}{\sec\theta}\,d\theta = \frac{3}{16}\int_0^{\frac{\pi}{3}} \frac{\sin^3\theta}{\cos^2\theta}\,d\theta$$

$$= \frac{3}{16}\int_0^{\frac{\pi}{3}} \frac{1-\cos^2\theta}{\cos^2\theta}\, \sin\theta \, d\theta$$

let $w = \cos\theta \Rightarrow dw = -\sin\theta \, d\theta$

$$= -\frac{3}{16}\int_1^{\frac{1}{2}} \frac{1-u^2}{u^2}\, du = \frac{3}{32}.$$

<u>EX7</u> Evaluate $\displaystyle\int \frac{x}{\sqrt{3-2x-x^2}}\, dx$

97

$$\int \frac{x}{\sqrt{3-2x-x^2}}\, dx = \int \frac{x \quad dx}{\sqrt{-(x^2+2x-3)}}$$

$$= \int \frac{x \quad dx}{\sqrt{-(x^2+2x+1)+4}}$$

$$= \int \frac{x \quad dx}{\sqrt{4-(x+1)^2}}.$$

$u = x+1 \Rightarrow du = dx$

$$= \int \frac{u-1}{\sqrt{4-u^2}}\, du$$

let $u = 2\sin\theta$

$du = 2\cos\theta \, d\theta$

$$= \int \frac{2\sin\theta - 1}{2\cos\theta}\, 2\cos\theta \, d\theta = -2\cos\theta - \theta + C$$

$$= -\sqrt{4-u^2} - \sin^{-1}\left(\frac{u}{2}\right) + C$$

$$= -\sqrt{3-2x-x^2} - \sin^{-1}\left(\frac{x+1}{2}\right) + C$$

EFS :

Q34 Find the Area of the region bounded by the hyperbola $9x^2 - 4y^2 = 36$ and the line $x = 3$

END.