6.2 Volume by Cylindrical Shell

Why do we need this Method?

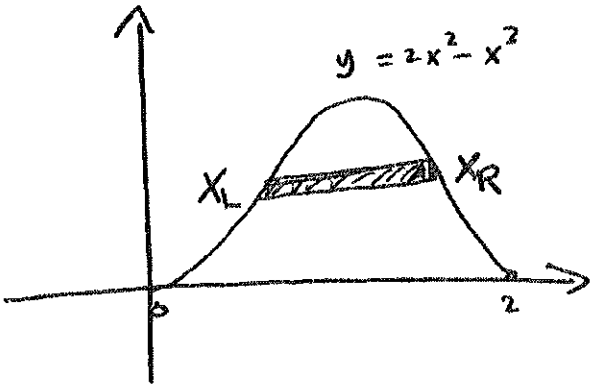
Answer: because some volume problems are difficult if we used the method in 6.1

Ex Find the volume of the solid obtained by rotating the region bounded by $y = 2x^2 - x^3$ and $y = 0$ about the y-axis.

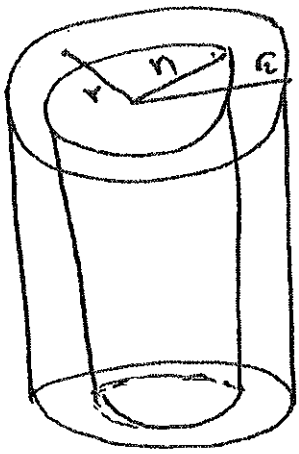
thickness : Δy

outer radius = ?

inner radius = ?



Method of cylindrical shell



$$V = V_2 - V_1$$

$$= \pi r_2^2 h - \pi r_1^2 h$$

$$= \pi (r_2^2 - r_1^2) h$$

$$= \pi (r_2 + r_1)(r_2 - r_1) h$$

$$= 2\pi \frac{(r_2 + r_1)}{2} (r_2 - r_1) h$$

$$= 2\pi r h \Delta r$$

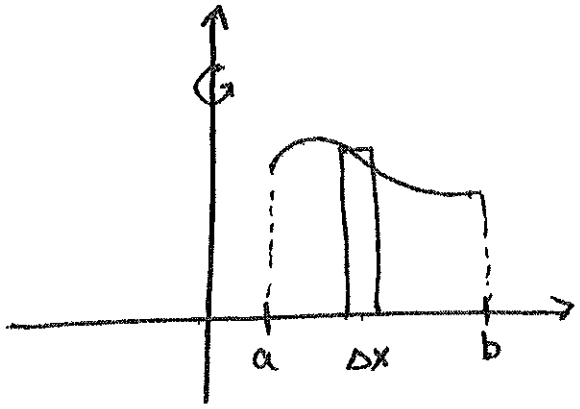
where $\Delta r = r_2 - r_1$

$r = \frac{r_1 + r_2}{2}$.

Remark: $V = 2\pi r\, h\, \Delta r$

$= [\text{Circumference}][\text{height}][\text{thickness}]$

The idea

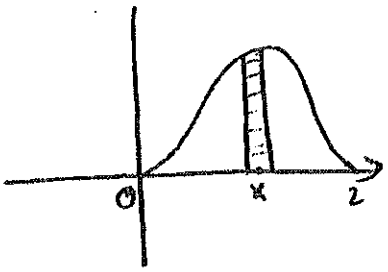


$$V_i = (2\pi \bar{x}_i)\,[f(\bar{x}_i)]\,\Delta x$$

$$V \approx \sum_{i=1}^{n} V_i = \sum_{i=1}^{n} 2\pi \bar{x}_i\, f(\bar{x}_i)\,\Delta x$$

$$V = \lim_{n\to\infty} \sum_{i=1}^{n} 2\pi \bar{x}_i\, f(\bar{x}_i)\,\Delta x = \int_a^b 2\pi x\, f(x)\,dx$$

Ex1 Find the volume of the solid obtained by rotating about the y-axis the region bounded by $y = 2x^2 - x^3$ and $y = 0$



average radius $= x$

Circumference $= 2\pi x$

height $= 2x^2 - x^3$

thickness $= \Delta x$

$$V = \int_0^2 (2\pi x)(2x^2 - x^3)\,dx = 2\pi \int_0^2 (2x^3 - x^4)\,dx$$

$$= \frac{16}{5}\pi .$$

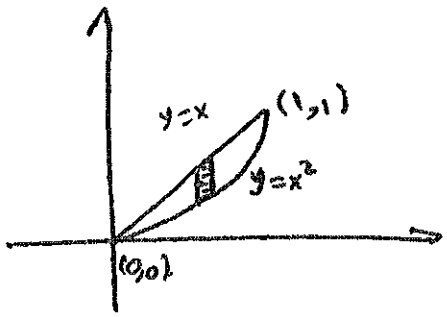
EX2 Find the volume of the solid obtained by rotating about the y-axis the region between $y = x$ and $y = x^2$.

thickness $= \Delta x$

Av. radius $= x$

height $= x - x^2$

Circumference $= 2\pi x$



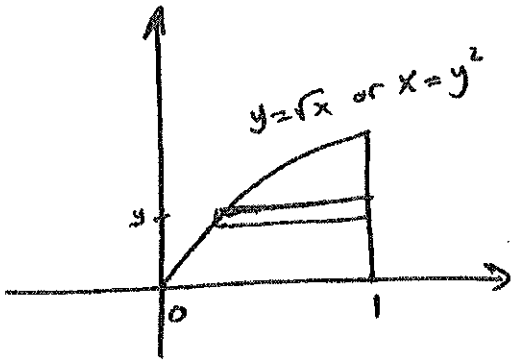
$$V = \int_0^1 (2\pi x)(x - x^2)\,dx = \frac{\pi}{6} .$$

EX3 Use cylindrical shell to find the volume of the solid obtained by rotating about the x-axis the region under the curve $y = \sqrt{x}$ from 0 to 1

thickness = Δy

Avarage radius = y

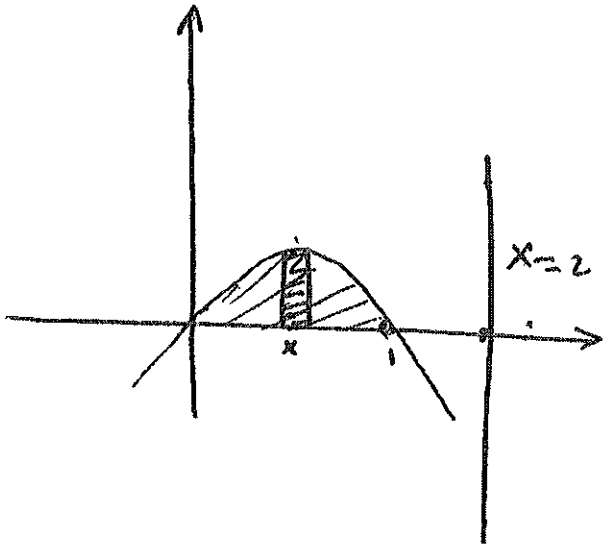
circumference = $2\pi y$



height = $1-y^2$

$$V = \int_0^1 (2\pi y)(1-y^2)\,dy = \frac{\pi}{2}.$$

EX4 Find the volume of the solid obtained by rotating the region bounded by $y = x - x^2$ and $y = 0$ about the Line $x = 2$.



thickness = Δx

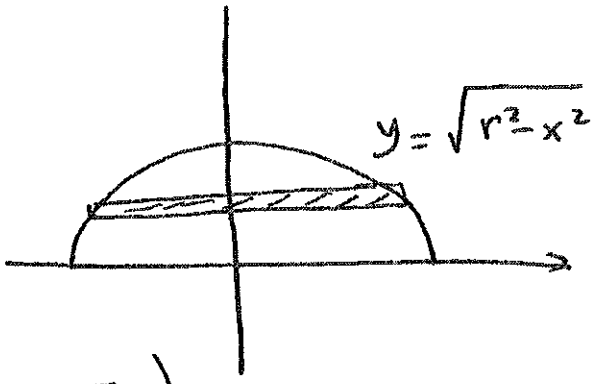
Avarage radius = $2 - x$

circumference = $2\pi(2-x)$

height = $x - x^2$

$$V = \int_0^1 2\pi(2-x)(x-x^2)\,dx = \frac{\pi}{2}.$$

Use cylindrical shell to find the volume of A sphere of radius r.



thickness $= \Delta y$

radius $= y$

circumference $= 2\pi y$

height $= \sqrt{r^2-y^2} - (-\sqrt{r^2-y^2})$

$= 2\sqrt{r^2-y^2}$

$$V = \int_0^r 2\pi y \left(2\sqrt{r^2-y^2}\right) dy = \frac{4}{3}\pi r^3.$$

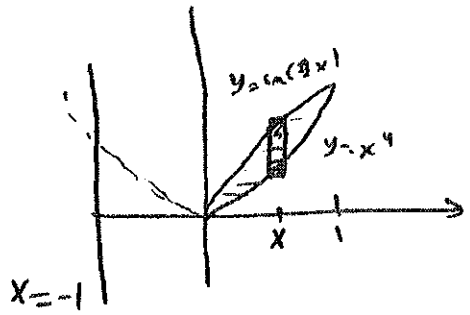
Set up, but do not evaluate, an integral for the volume of the solid obtained by rotating the region bounded by $y = x^4$ and $y = \sin(\frac{\pi}{2}x)$ about the line $x = -1$.

thickness $= \Delta x$

radius $= x+1$

circumference $= 2\pi(x+1)$

height $= \sin(\frac{\pi}{2}x) - x^4$



$$V = \int_0^1 2\pi(x+1)\left[\sin(\tfrac{\pi}{2}x) - x^4\right] dx$$

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END.