(A)

King Fahd University of Petroleum and Minerals

Department of Mathematics

Math - 106 Semester - 251 Quiz # III

Name: **S. No.:** **ID:**

Maximum Marks: 05 Section: Time Allowed: 15 Minutes

Instructions: *There are Five multiple choice questions. Each question carry equal mark. Put the right sign ($\surd$) against the correct answer. Give the answer of all questions.*

---

1. If $y = x^x$, then the relative rate of change of y with respect to x at the point $x = 1$ is

   $y = x^x \Rightarrow \ln y = \ln x^x = x \ln x \Rightarrow \frac{1}{y} \cdot \frac{dy}{dx} = x \cdot \frac{1}{x} + (\ln x) \cdot 1$

   $\frac{dy}{dx} = x^x (1 + \ln x) \Rightarrow$ relative rate of change of y at $x = 1$ is

   $\left.\frac{dy}{dx}\right|_{x=1} = 1^1 (1 + \ln 1) = 1 + 0 = 1$

   (a) 0
   (b) 1 ✓
   (c) 2
   (d) 3

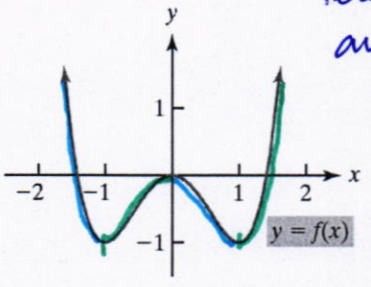
2. The graph of the function $y = f(x)$ is given:

   decreasing on $(-\infty, -1)$ and $(0, 1)$

   increasing on $(-1, 0)$ and $(1, \infty)$

   relative minimum at $(-1, -1)$ and $(1, -1)$

   relative maximum at $(0, 0)$



   Which of the following is true?

   (a) decreasing on $(-\infty, 0)$; increasing on $(0, \infty)$; no relative maximum
   (b) decreasing on $(-\infty, -1)$; increasing on $(-1, 0)$ and $(1, \infty)$; no relative extremum
   (c) decreasing on $(-\infty, -1)$; increasing on $(-1, 0)$; relative minima at $(-1, -1)$ ✓
   (d) decreasing on $(-\infty, -1)$; increasing on $(-1, \infty)$; relative maximum at $(0, 0)$

3. If $c = 0.2q^2 + 2q + 500$ is a cost function, then the marginal cost is changing, when $q = 97.357$, with the rate

marginal cost $= \frac{dc}{dq} = 0.4q + 2$

rate of change in marginal cost $= \frac{d}{dq}\left(\frac{dc}{dq}\right) = \frac{d^2c}{dq^2} = 0.4$

When $q = 97.357$, $\frac{d^2c}{dq^2} = 0.4$

(a) 0.2
(b) 2
(c) 97.357
(d) 0.4 ✓

4. If M and m are absolute maximum and absolute minimum, respectively, of the function $f(x) = x^{2/3}$ over the interval $[-8, 8]$, then $M \times m$ is

(a) 0 ✓
(b) 2/3
(c) 8
(d) 4

f is continuous on $[-8,8]$ and $f'(x) = \frac{2}{3}x^{-\frac{1}{3}}$

The only critical value on $(-8,8)$ is $x = 0$

$f(-8) = f(8) = 4$ and $f(0) = 0$. So, absolute minimum is 0 at $x = 0$ and absolute maximum is 4 at $x = -8, 8$.

Therefore, $M \times m = 4 \times 0 = 0$

5. If the cost function is $c = q^2 + 2q + 1$, then the average cost function $\bar{c}$ for $q > 0$ is

(a) concave up
(b) concave down
(c) both concave up as well concave down
(d) None of the above

average cost function $\bar{c} = \frac{c}{q} = q + 2 + \frac{1}{q}$

$\frac{d\bar{c}}{dq} = 1 - \frac{1}{q^2}$, and $\frac{d^2\bar{c}}{dq^2} = \frac{2}{q^3} > 0$ for $q > 0$

The graph of average cost function is concave up for $q > 0$

For Rough Work