MATH-101-10 term 171 Quiz 7

Name:- ID:-

Q1. Use logarithmic differentiation to find $\frac{dy}{dx}$ *If* $y=\sqrt[3]{\frac{(x^2+5)\cos^4 2x}{(x^3-8)^2}}$

$\ln y = \frac{1}{3}\left[\ln(x^2+5) + 4\ln\cos 2x - 2\ln(x^3-8)\right]$

$\frac{y'}{y} = \frac{1}{3}\left[\frac{2x}{x^2+5} + \frac{4(-2)\sin 2x}{\cos 2x} - \frac{2(3x^2)}{x^3-8}\right]$

$y' = \sqrt[3]{\frac{(x^2+5)\cos^4 2x}{(x^3-8)^2}}\left[\frac{1}{3}\left(\frac{2x}{x^2+5} - \frac{8\sin 2x}{\cos 2x} - \frac{6x^2}{x^3-8}\right)\right]$

Q2. Find $\frac{dy}{dx}$ if $y = x^{\log_{10} x}$

$\ln y = \log_{10} x \ln x$

$\frac{y'}{y} = \frac{\ln x}{x\ln 10} + \frac{\log_{10} x}{x}$

$y' = x^{\log_{10} x}\left[\frac{\ln x}{x\ln 10} + \frac{\log_{10} x}{x}\right]$

Q3. Oil spilled from a ruptured tanker spreads in a circle whose area increases at a constant rate of $10\, mi^2/hr$ .how fast is the radius of the spill increasing when the area is $15\, mi^2$?

$A = \pi r^2$ $\quad \frac{dA}{dt} = 10\ mi^2/hr$

find $\frac{dr}{dt}$ when $A = 15$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

find r if $A = 15$

$15 = \pi r^2$

$r = \sqrt{\frac{15}{\pi}}$

$\Rightarrow 10 = 2\pi\sqrt{\frac{15}{\pi}}\frac{dr}{dt}$

$\frac{dr}{dt} = \frac{5}{\pi}\sqrt{\frac{\pi}{15}}\ mi/hr$