

1. Name:

ID #:

(Show your work)

- Q1. Consider the function $y = f(x) = \underline{x^{\frac{2}{3}}(x-5)}$ Follow the steps to sketch the graph of the function.

- 1) Find the domain and symmetry if any

$$D = \mathbb{R} \quad \text{no symmetry}$$

- 2) Find y-int. then x-int. then check if the graph above the x-axis or below.

$$f \begin{array}{c} - \\ \hline - & 0 & - & 1 & 5 & + \end{array}$$

- 3) Find relative extreme then check if the graph increasing or decreasing (Check for vertical tangent at $x=0$)

$$y' = \frac{2}{3}x^{-\frac{1}{3}}(x-5) + x^{\frac{2}{3}} = x^{\frac{1}{3}} \left[\frac{2}{3}(x-5) + \frac{3x}{3} \right] = \frac{5x-10}{3\sqrt[3]{x}}$$

$$y' = 0 \Rightarrow x = \frac{10}{5} = 2$$

$$y' \text{ DNE } x = 0$$

Vertical tangent at $x=0$

$$f' \begin{array}{c} + \\ \nearrow \\ 0 \end{array} \begin{array}{c} - \\ \searrow \\ 2 \end{array} \begin{array}{c} + \\ \nearrow \\ 2 \end{array}$$

Local max at $x=0$ Local min at $x=2$

- 4) Find asymptotes if any (Check the behavior of the graph as $x \rightarrow \infty$ and $x \rightarrow -\infty$)

$$\lim_{x \rightarrow -\infty} f(x) = -\infty \quad \lim_{x \rightarrow \infty} f(x) = \infty \quad \text{No asymptotes}$$

- 5) Find inflection points if any then check if the graph concave up or down

$$f'' = \frac{5(3\sqrt[3]{x}) - (5x-10)3\sqrt[3]{x}^2}{9\sqrt[3]{x^2}}$$

$$f'' \begin{array}{c} - \\ \nearrow \\ 1 \end{array} \begin{array}{c} + \\ \searrow \\ 0 \end{array} \begin{array}{c} + \\ \nearrow \\ 1 \end{array}$$

$$f'' = \frac{5}{3}x^{-\frac{1}{3}} - \frac{(5x-10)}{9}x^{-\frac{5}{3}} = \frac{10(x+1)}{9\sqrt[3]{x^4}}$$

IP.

y

- 6) Sketch f

