For type (a), use the power rule. For type (b), use the differentiation formula for exponential functions. [If $b \neq e$, first convert $b^{f(x)}$ to an e^u function.] For type (c), use logarithmic differentiation or first convert to an e^u function. Do not apply a rule in a situation where the rule does not apply. For example, the power rule does not apply to x^x.

PROBLEMS 12.5

In Problems 1–12, find y' by using logarithmic differentiation.

1. $y = (x+1)^2(x-2)(x^2+3)$
2. $y = (3x+4)(8x-1)^2(3x^2+1)^4$
3. $y = (3x^3-1)^2(2x+5)^3$
4. $y = (2x^2+1)\sqrt{8x^2-1}$
5. $y = \sqrt{x+1}\sqrt{x-1}\sqrt{x^2+1}$
6. $y = (2x+1)\sqrt{x^3+2}\sqrt[3]{2x+5}$
7. $y = \dfrac{\sqrt{1-x^2}}{1-2x}$
8. $y = \sqrt{\dfrac{x^2+5}{x+9}}$
9. $y = \dfrac{(2x^2+2)^2}{(x+1)^2(3x+2)}$
10. $y = \dfrac{x^2(1+x^2)}{\sqrt{x^2+4}}$
11. $y = \sqrt{\dfrac{(x+3)(x-2)}{2x-1}}$
12. $y = \sqrt[3]{\dfrac{6(x^3+1)^2}{x^6e^{-4x}}}$

In Problems 13–20, find y'.

13. $y = x^{x^2+1}$
14. $y = (2x)^{\sqrt{x}}$
15. $y = x^{\sqrt{x}}$
16. $y = \left(\dfrac{3}{x^2}\right)^x$
17. $y = (3x+1)^{2x}$
18. $y = (x^2+1)^{x+1}$
19. $y = 4e^x x^{3x}$
20. $y = (\sqrt{x})^x$
21. If $y = (4x-3)^{2x+1}$, find dy/dx when $x = 1$.
22. If $y = (\ln x)^{\ln x}$, find dy/dx when $x = e$.
23. Find an equation of the tangent line to
$$y = (x+1)(x+2)^2(x+3)^2$$
at the point where $x = 0$.
24. Find an equation of the tangent line to the graph of
$$y = x^x$$
at the point where $x = 1$.
25. Find an equation of the tangent line to the graph of
$$y = x^x$$
at the point where $x = e$.
26. If $y = x^x$, find the relative rate of change of y with respect to x when $x = 1$.
27. If $y = (3x)^{-2x}$, find the value of x for which the *percentage* rate of change of y with respect to x is 60.
28. Suppose $f(x)$ is a positive differentiable function and g is a differentiable function and $y = (f(x))^{g(x)}$. Use logarithmic differentiation to show that
$$\frac{dy}{dx} = (f(x))^{g(x)}\left(f'(x)\frac{g(x)}{f(x)} + g'(x)\ln(f(x))\right)$$
29. The demand equation for a compact disc is
$$q = 500 - 40p + p^2$$
If the price of \$15 is increased by 1/2%, find the corresponding percentage change in revenue.
30. Repeat Problem 29 with the same information except for a 5% *decrease* in price.

Objective

To approximate real roots of an equation by using calculus. The method shown is suitable for calculators.

12.6 Newton's Method

It is easy to solve equations of the form $f(x) = 0$ when f is a linear or quadratic function. For example, we can solve $x^2+3x-2 = 0$ by the quadratic formula. However, if $f(x)$ has a degree greater than 2 (or is not a polynomial), it may be difficult, or even impossible, to find solutions (or roots) of $f(x) = 0$ by the methods to which you are accustomed. For this reason, we may settle for approximate solutions, which can be obtained in a variety of efficient ways. For example, a graphing calculator can be used to estimate the real roots of $f(x) = 0$. In this section, we will study how the derivative can be so used (provided that f is differentiable). The procedure we will develop, called *Newton's method,* is well suited to a calculator or computer.

Newton's method requires an initial estimate for a root of $f(x) = 0$. One way of obtaining this estimate is by making a rough sketch of the graph of $y = f(x)$ and estimating the root from the graph. A point on the graph where $y = 0$ is an x-intercept, and the x-value of this point is a root of $f(x) = 0$. Another way of locating a root is based on the following fact:

> If f is continuous on the interval $[a, b]$ and $f(a)$ and $f(b)$ have opposite signs, then the equation $f(x) = 0$ has at least one real root between a and b.

Figure 12.6 depicts this situation. The x-intercept between a and b corresponds to a root of $f(x) = 0$, and we can use either a or b to approximate this root.