PROBLEMS 12.4

In Problems 1–24, find dy/dx by implicit differentiation.

1. $x^2 + 4y^2 = 4$
2. $3x^2 + 6y^2 = 1$
3. $2y^3 - 7x^2 = 5$
4. $5y^2 - 2x^2 = 10$
5. $\sqrt[3]{x} + \sqrt[3]{y} = 3$
6. $x^{1/5} + y^{1/5} = 4$
7. $x^{3/4} + y^{3/4} = 5$
8. $y^3 = 4x$
9. $xy = 36$
10. $x^2 + xy - 2y^2 = 0$
11. $xy - y - 11x = 5$
12. $x^3 - y^3 = 3x^2y - 3xy^2$
13. $2x^3 + y^3 - 12xy = 0$
14. $5x^3 + 6xy + 7y^3 = 0$
15. $x = \sqrt{y} + \sqrt[4]{y}$
16. $x^3y^3 + x = 9$
17. $5x^3y^4 - x + y^2 = 25$
18. $y^2 + y = \ln x$
19. $\ln(xy) = e^{xy}$
20. $\ln(xy) + x = 4$
21. $xe^y + y = 13$
22. $4x^2 + 9y^2 = 16$
23. $(1 + e^{3x})^2 = 3 + \ln(x + y)$
24. $e^{x-y} = \ln(x - y)$
25. If $x + xy + y^2 = 7$, find dy/dx at (1, 2).
26. If $x\sqrt{y+1} = y\sqrt{x+1}$, find dy/dx at (3, 3).
27. Find the slope of the curve $4x^2 + 9y^2 = 1$ at the point $\left(0, \frac{1}{3}\right)$; at the point (x_0, y_0).
28. Find the slope of the curve $(x^2 + y^2)^2 = 4y^2$ at the point (0, 2).
29. Find equations of the tangent lines to the curve
$$x^3 + xy + y^3 = -1$$
at the points $(-1, -1)$, $(-1, 0)$, and $(-1, 1)$.
30. Repeat Problem 29 for the curve
$$y^2 + xy - x^2 = 5$$
at the point (4, 3).

For the demand equations in Problems 31–34, find the rate of change of q with respect to p.

31. $p = 100 - q^2$
32. $p = 400 - \sqrt{q}$
33. $p = \dfrac{20}{(q+5)^2}$
34. $p = \dfrac{3}{q^2 + 1}$
35. **Radioactivity** The relative activity I/I_0 of a radioactive element varies with elapsed time according to the equation
$$\ln\left(\frac{I}{I_0}\right) = -\lambda t$$
where λ (a Greek letter read "lambda") is the disintegration constant and I_0 is the initial intensity (a constant). Find the rate of change of the intensity I with respect to the elapsed time t.
36. **Earthquakes** The magnitude M of an earthquake and its energy E are related by the equation[6]
$$1.5M = \log\left(\frac{E}{2.5 \times 10^{11}}\right)$$
Here M is given in terms of Richter's preferred scale of 1958 and E is in ergs. Determine the rate of change of energy with respect to magnitude and the rate of change of magnitude with respect to energy.
37. **Physical Scale** The relationship among the speed (v), frequency (f), and wavelength (λ) of any wave is given by
$$v = f\lambda$$
Find $df/d\lambda$ by differentiating implicitly. (Treat v as a constant.) Then show that the same result is obtained if you first solve the equation for f and then differentiate with respect to λ.
38. **Biology** The equation $(P + a)(v + b) = k$ is called the "fundamental equation of muscle contraction."[7] Here P is the load imposed on the muscle, v is the velocity of the shortening of the muscle fibers, and a, b, and k are positive constants. Use implicit differentiation to show that, in terms of P,
$$\frac{dv}{dP} = -\frac{k}{(P+a)^2}$$
39. **Marginal Propensity to Consume** A country's savings S is defined implicitly in terms of its national income I by the equation
$$S^2 + \frac{1}{4}I^2 = SI + I$$
where both S and I are in billions of dollars. Find the marginal propensity to consume when $I = 16$ and $S = 12$.
40. **Technological Substitution** New products or technologies often tend to replace old ones. For example, today most commercial airlines use jet engines rather than prop engines. In discussing the forecasting of technological substitution, Hurter and Rubenstein[8] refer to the equation
$$\ln\frac{f(t)}{1 - f(t)} + \sigma\frac{1}{1 - f(t)} = C_1 + C_2 t$$
where $f(t)$ is the market share of the substitute over time t and C_1, C_2, and σ (a Greek letter read "sigma") are constants. Verify their claim that the rate of substitution is
$$f'(t) = \frac{C_2 f(t)[1 - f(t)]^2}{\sigma f(t) + [1 - f(t)]}$$

Objective

To describe the method of logarithmic differentiation and to show how to differentiate a function of the form u^v.

12.5 Logarithmic Differentiation

A technique called **logarithmic differentiation** often simplifies the differentiation of $y = f(x)$ when $f(x)$ involves products, quotients, or powers. The procedure is as

[6] K. E. Bullen, *An Introduction to the Theory of Seismology* (Cambridge, U.K.: Cambridge at the University Press, 1963).

[7] R. W. Stacy et al., *Essentials of Biological and Medical Physics* (New York: McGraw-Hill Book Company, 1955).

[8] A. P. Hurter, Jr., A. H. Rubenstein et al., "Market Penetration by New Innovations: The Technological Literature," *Technological Forecasting and Social Change*, 11 (1978), 197–221.