King Fahd University of Petroleum and Minerals
Department of Mathematics

Math 106

Major Exam II

241

November 18 , 2024

Net Time Allowed: 90 Minutes

MASTER VERSION

S. 12.5
P 13

1. If $y = x^{x^2+1}$, then $y' =$

(a) $x^{x^2}\left(x^2+1+2x^2\ln x\right)$ ________________________________(correct)

(b) $x^{x^2+1}\left(x^2+1+2x^2\ln x\right)$

(c) $x^{x^2}\left(x^2+1+x\ln x\right)$

(d) $x^{x^2+1}\left(\frac{x^2+1}{x}+\ln x\right)$

(e) $x^{x^2+1}\left(\frac{x^2+1}{x^2}+2x\ln(x+1)\right)$

By using logarithmic differentiation

$\ln y = \ln x^{x^2+1}$

$\Rightarrow \ln y = (x^2+1)\ln x$

$\frac{1}{y}y' = 2x\ln x + (x^2+1)\frac{1}{x}$

$\Rightarrow y' = y\left[2x\ln x + \frac{(x^2+1)}{x}\right]$

$y' = x^{x^2+1}\left[2x\ln x + \frac{(x^2+1)}{x}\right]$

$y' = x^{x^2}\left[2x^2\ln x + x^2+1\right]$.

S 12.7
P 37

2. If $c = 0.2q^2 + 2q + 500$ is a cost function, how fast is the marginal cost changing when $q = 95.5436$?

(a) 0.4 ________________________________(correct)

(b) 0.2

(c) 0.1

(d) 0.3

(e) 0.6

$C'(q) = 0.4q + 2$

$C''(q) = 0.4$

when $q = 95.5436 \longrightarrow C''(q) = 0.4$.

S:12.7 P:20

3. If $y = \dfrac{x}{e^x}$, then $y'' =$

(a) $\dfrac{x-2}{e^x}$ ______ (correct)

(b) $\dfrac{x+2}{e^x}$

(c) $\dfrac{x-1}{e^x}$

(d) $\dfrac{x-1}{e^{2x}}$

(e) $\dfrac{x}{e^{2x}}$

$$* \quad y' = \frac{(1)e^x - xe^x}{(e^x)^2}$$

$$= \frac{e^x(1-x)}{e^{2x}} = \frac{1-x}{e^x}$$

$$* \quad y'' = \frac{(-1)e^x - (1-x)e^x}{(e^x)^2}$$

$$= \frac{e^x(x-2)}{e^{2x}} = \frac{x-2}{e^x}.$$

similar S.13.1 P.71

4. For a manufacturer's product, the revenue function is given by $r = 12q + 2q^2 - \dfrac{q^3}{3}$. Then, r is maximum when the output is

(a) 6 ______ (correct)

(b) 4

(c) 2

(d) −2

(e) −6

$r' = 12 + 4q - q^2$

to find the critical values. $\rightarrow r' = 0$

$12 + 4q - q^2 = 0$

$-(q^2 - 4q - 12) = 0$

$-(q-6)(q+2) = 0$ since $q \geq 0 \Rightarrow \boxed{q = 6}$

0 6 ∞

$r'(q)$: $(-)(-)(+) = (+)$ $(-)(+)(+) = (-)$

increasing decreasing

$\therefore$ r is maximum when $q = 6$

similar 5.13.1 P.30

5. The function $y = \sqrt[3]{x}$ is

(a) increasing on the interval $(-\infty, 0) \cup (0, \infty)$ ______ (correct)
(b) increasing on the interval $(-\infty, \infty)$
(c) decreasing on the interval $(-\infty, 0)$
(d) decreasing on the interval $(0, \infty)$
(e) decreasing on the interval $(-\infty, 0) \cup (0, \infty)$

$y = \sqrt[3]{x} = x^{1/3}$ (Domain: $\mathbb{R}$)

$y' = \frac{1}{3} x^{-2/3} = \frac{1}{3\sqrt[3]{x^2}}$

The critical point is: $x = 0$

$-\infty$ ——— 0 ——— ∞

$y'(x)$: $(+)$ increasing, $(+)$ increasing

$\therefore$ y is increasing on $(-\infty, 0) \cup (0, \infty)$. ■

similar 5.13.2 P 9

6. If M and m are the absolute maximum and absolute minimum of the function $f(x) = 2x^2 - x^4$ on the interval $[-1, 2]$, then $M + m =$

(a) -7 ______ (correct)
(b) -8
(c) 1
(d) 2
(e) 0

Domain: $\mathbb{R}$.

$f'(x) = 4x - 4x^3$

* $f'(x) = 0$

$4x(1 - x^2) = 0$

The critical points are: $x = 0$, $x = -1$, $x = 1$

* $f(-1) = 2(-1)^2 - (-1)^4 = 2 - 1 = 1 \to$ maximum

$f(0) = 0$

$f(1) = 2(1)^2 - (1)^4 = 2 - 1 = 1 \to$ maximum

$f(2) = 2(2)^2 - (2)^4 = 8 - 16 = -8 \to$ minimum

$\Rightarrow$ $1 - 8 = -7$. ■

S. 13.3 / P. 31

7. Which of the following statements is **FALSE** for the function $f(x) = xe^x$.

(a) always concave up ________ (correct)

(b) has one critical point

(c) has one inflection point

(d) increasing in $(-1, \infty)$

(e) concave downward in $(-\infty, -2)$

Domain: $\mathbb{R}$

$f'(x) = (1)e^x + xe^x = e^x(1+x)$

* f has only one critical value → (b) is true.

-1 ∞ $(+)$

* f is increasing on $(-1, \infty)$ → (d) is true.

$f''(x) = e^x(1+x) + e^x(1) = e^x(x+2)$

* f has only one inflection point → (c) is true.

$-\infty$ $(-)$ -2

* f is concave downward on $(-\infty, -2)$

→ (e) is true.

8. The function $f(x) = \dfrac{x^2+x+1}{x}$ concave downward on the interval

Similar S. 13.3 / P. 34

(a) $(-\infty, 0)$ ________ (correct)

(b) $(-\infty, 1)$

(c) $(0, \infty)$

(d) $(1, \infty)$

(e) $(-\infty, \infty)$

Domain: $\mathbb{R} - \{0\}$

* $f'(x) = \dfrac{(2x+1)x - (x^2+x+1)(1)}{x^2} = \dfrac{2x^2+x-x^2-x-1}{x^2}$

$= 1 - 1/x^2$

* $f''(x) = 2x^{-3}$

→ no inflection points.

$-\infty$ 0 ∞

$f''(x)$: $(-)$ $(+)$

concave downward | concave upward.

$\therefore$ f is concave downward on $(-\infty, 0)$

S. 13.4 / P. 5

9. If $y = \frac{1}{3}x^3 + 2x^2 - 5x + 1$ has a relative maximum when $x = a$ and a relative minimum when $x = b$, then $a + 5b =$

(a) 0 ________________________________ (correct)

(b) -2

(c) -4

(d) -3

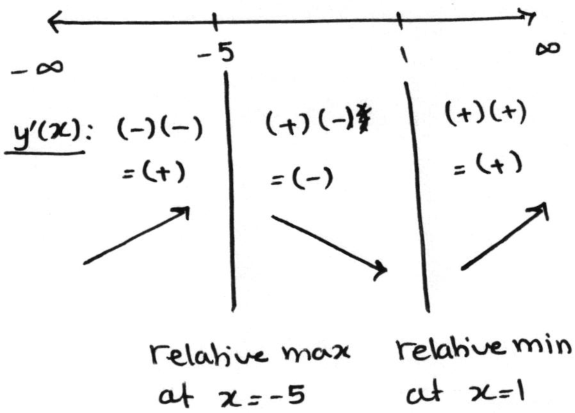
(e) -1

Domain: $\mathbb{R}$

$y' = x^2 + 4x - 5$

$\Rightarrow (x+5)(x-1) = 0$

$\boxed{x = -5}$ or $\boxed{x = 1}$



$a = -5$

$b = 1$

$a + 5b = -5 + 5 = 0$ ∎

Similar S. 13.5 / P. 23

10. The function $f(x) = 2e^{x-2} - 2$ has

(a) one horizontal asymptote ________________________________ (correct)

(b) two horizontal asymptotes

(c) one vertical asymptote

(d) no asymptote

(e) one vertical and one horizontal asymptote

Domain: $\mathbb{R}$

$\rightarrow$ No vertical asymptotes.

$\lim_{x \to -\infty} 2e^{x-2} - 2 = 2 - 2 = 0$

$\lim_{x \to \infty} 2e^{x-2} - 2 = \infty$

$\therefore$ only one horizontal asymptote. ∎

S. 13.6 / P. 13

11. For a monopolist's product, the demand equation is $p = 42 - 4q$ and the average-cost function is $\bar{c} = 2 + \dfrac{80}{q}$. The profit-maximizing price is

(a) 22 ______________________________ (correct)

(b) 42

(c) 5

(d) 8

(e) 62

Profit = Total Revenue − Total Cost

where Total Cost $c = \bar{c}q = 2q + 80$

$$P = pq - c = (42-4q)q - (2q+80)$$
$$= -(4q^2 - 40q + 80)$$

$P' = -(8q - 40) \Rightarrow P' = 0$ when $q = 5$

$P'' = -8 < 0 \Rightarrow$ so P is maximum when $q = 5$

Therefore, the price $p = 42 - 4(5) = 22$. ∎

S. 14.1 / P 25

12. Using differential $e^{0.001} \approx$

(a) 1.001 ______________________________ (correct)

(b) 1.01

(c) 0.01

(d) 0.001

(e) 1.1

Let $f(x) = e^x$

$f(x + dx) = f(x) + dy = e^x + e^x dx$

If $x = 0$ & $dx = 0.001$, then

$$e^{0.001} = f(0 + 0.001) = e^0 + e^0(0.001)$$
$$= 1 + 0.001 = 1.001$$ ∎

Similar S 14.2 P 35

13. $\int \left(u^{e+1} + e^{u+1} + \sqrt{u} + \sqrt{2}\right) du =$

(a) $\dfrac{u^{e+2}}{e+2} + e^{u+1} + \dfrac{2\sqrt{u^3}}{3} + \sqrt{2}u + C$ ________ (correct)

(b) $\dfrac{u^{e+1}}{e+1} + e^{u+1} + \dfrac{2\sqrt[3]{u^2}}{3} + \sqrt{2}u + C$

(c) $\dfrac{u^{e+1}}{e+1} + e^{u+2} + \dfrac{2\sqrt{u^3}}{3} + \sqrt{2}u + C$

(d) $\dfrac{u^{e}}{e+1} + e^{u+2} + \dfrac{3\sqrt[3]{u^2}}{2} + \sqrt{2}u^2 + C$

(e) $\dfrac{u^{e+2}}{e+2} + e^{u+2} + \dfrac{2\sqrt{u^3}}{3} + \sqrt{2}u^2 + C$

$\int u^{e+1}du + \int e^{\boxed{u+1}}du + \int u^{1/2}du + \int \sqrt{2}\,du$

let $v = u+1$
$dv = du$
$\int e^v dv = e^v = e^{u+1}$

$= \dfrac{u^{e+2}}{e+2} + e^{u+1} + \dfrac{2}{3}u^{3/2} + \sqrt{2}u + C$

Similar S 14.3 P 5

14. The solution of the differential equation $y'' = -3x^2 + 4x$ satisfying the condition $y'(0) = y(0) = 1$ is

(a) $y = -\dfrac{x^4}{4} + \dfrac{2x^3}{3} + x + 1$ ________ (correct)

(b) $y = -\dfrac{x^4}{4} + \dfrac{2x^3}{3} + x - 1$

(c) $y = -x^4 + \dfrac{2x^3}{3} + x$

(d) $y = -\dfrac{x^4}{4} + \dfrac{3x^3}{2} + x + 2$

(e) $y = -\dfrac{x^3}{3} + \dfrac{2x^2}{2} + x - 2$

$y' = \int (-3x^2 + 4x)\,dx$

$= -x^3 + 2x^2 + C$

Given $y'(0) = 1 \Rightarrow 0 + 0 + C = 1$

$\Rightarrow \boxed{C=1}$

$\therefore y' = -x^3 + 2x^2 + 1$

$y = \int (-x^3 + 2x^2 + 1)\,dx$

$= \dfrac{-x^4}{4} + \dfrac{2}{3}x^3 + x + C$

Given $y(0) = 1 \Rightarrow 0 + 0 + 0 + C = 1$

$\Rightarrow \boxed{C=1}$

$\therefore y = \dfrac{-x^4}{4} + \dfrac{2}{3}x^3 + x + 1$

S 14.3
P. 11

15. The marginal-revenue of a product is $\frac{dr}{dq} = 275 - q - 0.3q^2$ Then the unit price when producing 10 units $p(10) =$

(a) 260 ________________ (correct)

(b) 240

(c) 200

(d) 1260

(e) 2800

Since $r = pq \Rightarrow p = \frac{r}{q}$

$$r = \int (275 - q - 0.3q^2)\, dq$$

$$= 275q - \frac{q^2}{2} - \frac{1}{10}q^3 + C$$

If $q = 0 \Rightarrow r = 0$. Hence $0 + C = 0$ $\boxed{C = 0}$

$$\therefore r = 275q - \frac{q^2}{2} - \frac{1}{10}q^3$$

$$p = \frac{1}{q} r = \frac{1}{q}\left[275q - \frac{q^2}{2} - \frac{1}{10}q^3\right]$$

$$= 275 - \frac{q}{2} - \frac{1}{10}q^2$$

$$\therefore P(10) = 275 - \frac{10}{2} - \frac{10^2}{10}$$

$$= 260$$