1. Let

$$f(x) = \begin{cases} \sqrt{2-x} & \text{if } x < 2, \\ x^3 + k(x+1) & \text{if } x \geq 2. \end{cases}$$

Determine the value of k for which $\lim_{x \to 2} f(x)$ exists.

exact problem 64. Section 10.2

$\lim_{x\to 2} f(x)$ exists $\iff \lim_{x\to 2^+} f(x) = \lim_{x\to 2^-} f(x)$

(a) $-\frac{8}{3}$

(b) 0

(c) -8

(d) $\frac{5}{3}$

(e) $-\frac{4}{3}$

$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} (x^3 + k(x+1)) = 8+3k$

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} \sqrt{2-x} = 0$

$8+3k=0$

$k = -\frac{8}{3}$

2. If $\lim_{t\to\infty} \frac{4t^n + t^{n-1} - 2}{7t^m - 2t^{m-1} + 28} = \frac{4}{7}$, then $m - n =$

similar to problem 25. Section 10.2

if $m=n$

$\lim_{t\to\infty} \frac{4t^n + t^{n-1} - 2}{7t^m - 2t^{m-1} + 28} = \frac{4}{7}$

(a) 0

(b) 1

(c) -1

(d) 2

(e) -2

then $m-n=0$

3. If $a, b > 0$ and the function

$$f(x) = \begin{cases} \dfrac{a}{x^2} & \text{if } x < -3 \\ bx + 1 & \text{if } x \geq -3 \end{cases}$$

is continuous at $x = -3$, then $\sqrt{a + 27b} =$

similar to problem 34. Section 10.3

$f(x)$ is continuous at $x=-3 \Leftrightarrow \lim_{x\to -3} f(x) = f(-3)$

$\boxed{\text{(a) } 3}$
(b) 1
(c) 9
(d) 6
(e) 18

$f(-3) = b(-3)+1 = -3b+1$

$\lim_{x\to -3^+} f(x) = \lim_{x\to -3^+} (bx+1) = -3b+1$

$\lim_{x\to -3^-} f(x) = \lim_{x\to -3^-} \left(\frac{a}{x^2}\right) = \frac{a}{9}$

$\Rightarrow \frac{a}{9} = -3b+1 \Rightarrow a+27 = 9$

$\sqrt{a+27} = 3$

4. The curve of $f(x) = x^3 - 6x^2 + 12x + 12$ has

similar to problem 35. Section 11.1

the curve of $f(x)$ has horizontal ~~ass~~ tangent at $x=x_0$ when $f'(x_0)=0$

$\boxed{\text{(a)}}$ one horizontal tangent line
(b) one vertical tangent line
(c) three horizontal tangent lines
(d) two horizontal tangent lines
(e) two vertical tangent lines

$f'(x) = 3x^2 - 12x + 12$

$3x^2 - 12x + 12 = 0$

$x^2 - 4x + 4 = 0$

$(x-2)^2 = 0$

$x - 2 = 0$

$x = 2 \Rightarrow$ one H.T.

$f(x)$ is continuous poly function

No V.T.

5. $\lim_{t \to 5} \frac{\sqrt{t+4}-3}{t-5} =$

similar to problem 64. Section 10.1

(a) $\boxed{\frac{1}{6}}$

(b) $\frac{1}{9}$

(c) $\frac{1}{3}$

(d) ∞

(e) 0

$$= \lim_{t \to 5} \frac{\sqrt{t+4}-3}{t-5} \cdot \frac{\sqrt{t+4}+3}{\sqrt{t+4}+3}$$

$$= \lim_{t \to 5} \frac{\cancel{t+4-9}}{(\sqrt{t+4}+3)\cancel{(t-5)}}$$

$$= \lim_{t \to 5} \frac{1}{\sqrt{t+4}+3} = \frac{1}{\sqrt{9}+3} = \frac{1}{6}$$

6. The average cost $\bar{c}$ for producing q units of a product is $\bar{c} = 0.4q + 4 + \frac{9}{q}$. The marginal cost when producing one unit is

similar to problem 32 and problems 19-22. Section 11.3

(a) $\boxed{4.8}$

(b) 4.2

(c) 4.4

(d) 4.1

(e) 4.6

$$c = q\bar{c}$$

$$= q\left(0.4q + 4 + \frac{9}{q}\right)$$

$$= 0.4q^2 + 4q + 9$$

$$\frac{dc}{dq} = 0.8q + 4$$

$$\frac{dc}{dq}(1) = 0.8 + 4 = 4.8$$

7. If the demand function for a product is $p(q) = \dfrac{q+5}{q+3}$, then the marginal-revenue when $q = 1$ is

similar to problem 62. Section 11.4

revenue $r = pq$

$$= \left(\frac{q+5}{q+3}\right) q$$

$$= \frac{q^2+5q}{q+3}$$

$$\frac{dr}{dq} = \frac{(2q+5)(q+3)-(q^2+5q)(1)}{(q+3)^2}$$

at $q=1$

$$\frac{dr}{dq}(1) = \frac{(7)(4)-(6)(1)}{(4)^2} = \frac{28-6}{16} = \frac{22}{16} = \frac{11}{8}$$

(a) $\boxed{\dfrac{11}{8}}$

(b) $\dfrac{1}{8}$

(c) $\dfrac{11}{2}$

(d) $\dfrac{21}{2}$

(e) $\dfrac{21}{8}$

8. The relative rate of change of $y = 2x^2 + 5$ at $x = 10$ is

exact problem 35. Section 11.3

relative rate of change $= \dfrac{y'}{y}$

$y' = 4x$

$$\frac{y'}{y} = \frac{4x}{2x^2+5}$$

$$\left.\frac{y'}{y}\right|_{x=10} = \frac{40}{205} = \frac{8}{41}$$

(a) $\boxed{\dfrac{8}{41}}$

(b) $\dfrac{8}{5}$

(c) $\dfrac{3}{11}$

(d) $\dfrac{1}{9}$

(e) $\dfrac{5}{22}$

9. If the demand equation is $p = 100 + aq + bq^2$ for some real numbers a and b such that

$$\frac{dq}{dp} = \begin{cases} 1 & \text{if } q = 0, \\ \frac{1}{2} & \text{if } q = \frac{1}{2}. \end{cases}$$

Then, $a + b =$

similar to problems 31-34. Section 12.4

$p = 100 + aq + bq^2$

take $\frac{d}{dp} \Rightarrow$

$1 = a\frac{dq}{dp} + 2bq\frac{dq}{dp}$.

at $q=0$, $\frac{dq}{dp} = 1 \Rightarrow 1 = a(1) + 0 \Rightarrow \boxed{a=1}$

at $q=\frac{1}{2}$ $\frac{dq}{dp} = \frac{1}{2} \Rightarrow 1 = (1)(\frac{1}{2}) + 2b(\frac{1}{2})(\frac{1}{2})$

$\frac{1}{2} = \frac{1}{2}b \Rightarrow \boxed{b=1}$

$a+b=2$

(a) 2

(b) 0

(c) -2

(d) $\frac{-1}{2}$

(e) $\frac{1}{2}$

10. A consumption function is given by

$$C = \frac{2\left(3\sqrt{I^3} + 2\right)}{I + 16}.$$

The marginal propensity to save when $I = 4$ is

similar to example 9. Section 11.4

$$\frac{dC}{dI} = \frac{9I^{3/2-1}(I+16) - 2(3\sqrt{I^3}+2)}{(I+16)^2}$$

when $I = 4$

$$\left.\frac{dc}{dI}\right|_{I=4} = \frac{9\sqrt{4}(20) - 2(3(2)^3+2)}{(20)^2} = \frac{360-52}{400} = \frac{77}{100}$$

the marginal propensity to consume

the marginal propensity to Save $= 1 - \frac{77}{100} = \frac{23}{100}$

(a) $\frac{23}{100}$

(b) $\frac{77}{100}$

(c) $\frac{11}{100}$

(d) $\frac{33}{100}$

(e) $\frac{73}{100}$

11. $\dfrac{d}{dx}\left(x\ln x - x\right) =$

exact problem 11. Section 12.1

$x(\frac{1}{x}) + \ln x - 1$

$\cancel{1} + \ln x - \cancel{1}$

$\ln x$

(a) $\ln x$

(b) $\dfrac{\ln x}{x}$

(c) $2 + \ln x$

(d) $x \ln x$

(e) $\ln x - 2$

12. The derivative of $y = e^{1+\sqrt{x}} + 5^{2x^3}$ is

exact problem 15+19. Section 12.2

$y' = e^{1+\sqrt{x}} \cdot \frac{d}{dx}(1+\sqrt{x}) + 5^{2x^3} \frac{d}{dx}(2x^3) \cdot \ln 5$

$= e^{1+\sqrt{x}} \frac{1}{2\sqrt{x}} + (6x^2)(5^{2x^3}) \ln 5$

(a) $\dfrac{e^{1+\sqrt{x}}}{2\sqrt{x}} + (6x^2)5^{2x^3}\ln 5$

(b) $\sqrt{x}e^{1+\sqrt{x}} + (2x^3 - 1)5^{2x^3}\ln 5$

(c) $\sqrt{x}e^{\sqrt{x}} + (2x^3 - 1)5^{2x^3-1}\ln 5$

(d) $\dfrac{e^{\sqrt{x}}}{2\sqrt{x}} + (6x^2)5^{2x^3-1}\ln 5$

(e) $\dfrac{e^{1+\sqrt{x}}}{2\sqrt{x}} + (6x^2 - 1)\dfrac{5^{2x^3-1}}{\ln 5}$

13. If $(1+e^{3x})^2 = 3 + \ln(x+y)$, then $y'(0,1) =$

exact problem 23. Section 12.4

$$2(1+e^{3x}) \cdot e^{3x} \cdot 3 = 3 + \frac{1+y'}{x+y}$$

- $\boxed{\text{(a) } 11}$
- (b) 12
- (c) 10
- (d) 9
- (e) 8

$$\left[6(1+e^{3x})e^{3x} \cdot 3\right](x+y) - 1 = y'$$

at $x=0$, $y=1$

$$y' = \left(6(1+e^0)e^0 \cdot 3\right)(1) - 1$$

$$= 18(2) - 1$$

$$= 12 - 1 = 11$$

14. If $y = 2u^3 + 3u^2 + 5u - 1$ and $u = 3x + 1$, then for $x = 0$, $dy/dx =$

exact problem 8. Section 11.5

- $\boxed{\text{(a) } 51}$
- (b) 41
- (c) 62
- (d) 71
- (e) 17

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= (6u^2 + 6u + 5)(3)$$

at $x = 0 \Rightarrow u = 1$

$$\left.\frac{dy}{dx}\right|_{u=1} = (17)(3) = 51$$

15. The slope of the curve $y = (x^2 - 7x - 8)^3$ at the ~~point (8, 0)~~ $x=8$ equal to

exact problem 57. Section 11.5

(a) 0

(b) 1

(c) -1

(d) -7

(e) ∞

$y' = 3(x^2-7x-8)^2(2x-7)$

at $x=8$

$y' = 3(\underbrace{64-56-8}_{=0})^2(16-7)$

$= 0$