EXAMPLE 3 **Using the Differential to Estimate a Change in a Quantity**

A governmental health agency examined the records of a group of individuals who were hospitalized with a particular illness. It was found that the total proportion P that are discharged at the end of t days of hospitalization is given by

$$P = P(t) = 1 - \left(\frac{300}{300 + t}\right)^3$$

Use differentials to approximate the change in the proportion discharged if t changes from 300 to 305.

Solution: The change in t from 300 to 305 is $\Delta t = dt = 305 - 300 = 5$. The change in P is $\Delta P = P(305) - P(300)$. We approximate ΔP by dP:

$$\Delta P \approx dP = P'(t)\,dt = -3\left(\frac{300}{300 + t}\right)^2\left(-\frac{300}{(300 + t)^2}\right)dt = 3\frac{300^3}{(300 + t)^4}\,dt$$

When $t = 300$ and $dt = 5$,

$$dP = 3\frac{300^3}{600^4}5 = \frac{15}{2^3 600} = \frac{1}{2^3 40} = \frac{1}{320} \approx 0.0031$$

For a comparison, the true value of ΔP is

$$P(305) - P(300) = 0.87807 - 0.87500 = 0.00307$$

(to five decimal places).