1. Evaluate $\lim_{x\to\infty} \frac{3x+5x^2-8}{x^2+4x-5}$. $= \lim_{x\to 1} \frac{5x^2}{x^2} = 5$

(a) 2

(b) 3

(c) 4

(d) $\frac{7}{3}$

✓ ~~(e)~~ 5

2. Evaluate $\lim_{t\to 9} \frac{\sqrt{t}-3}{t-9}$. $= \lim_{t\to 9} \frac{(\sqrt{t}-3)(\sqrt{t}+3)}{(t-9)(\sqrt{t}+3)} = \lim_{t\to 9} \frac{t-9}{(t-9)(\sqrt{t}+3)}$

$= \lim_{t\to 9} \frac{1}{\sqrt{t}+3} = \frac{1}{6}$

(a) $\frac{1}{3}$

✓ ~~(b)~~ $\frac{1}{6}$

(c) $\frac{1}{12}$

(d) 3

(e) $\frac{1}{18}$

3. Let

$$f(x) = \begin{cases} \sqrt{6-x}, & x < 2, \\ x^3 + k(x-1) + 1, & x \geq 2. \end{cases}$$

Find k so that f is continuous at $x = 2$.

(a) $k = 3$

(b) $k = -2$

✓ ~~(c)~~ $k = -7$

(d) $k = 0$

(e) $k = 2$

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} \sqrt{6-x}$

$= 2$

$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} (x^3+k(x-1)+1)$

$= 8 + k(1) + 1 = 9 + k$

Since f is continuous at $x=2$, $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$.

So, $2 = 9+k \Rightarrow k = -7$

4. Using the definition of derivative, for $f(x) = \dfrac{1}{x+a}$.

$f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \dfrac{\frac{1}{x+a+h} - \frac{1}{x+a}}{h}$

$= \lim_{h \to 0} \dfrac{x - \cancel{a} - x - \cancel{a} - h}{(x+a)(x+a+h)h}$

(a) $f'(x) = \lim_{h \to 0} \dfrac{x - x + h}{(ah + xh)(x + h + a)}$.

(b) $f'(x) = \lim_{h \to 0} \dfrac{x - x - h}{(ah + xh)(x + h + a)}$.

(c) $f'(x) = \lim_{h \to 0} \dfrac{x - x - h}{(a + x)(x + h + a)}$.

(d) $f'(x) = \lim_{h \to 0} \dfrac{h}{(a + x)(x + h + a)}$.

(e) $f'(x) = \lim_{h \to 0} \dfrac{x - x - h}{(ah + xh)}$.

5. For $y = 3x - 6\sqrt{x}$, the slope of the tangent line at $x = 9$ is:

$y' = 3 - \dfrac{6}{2\sqrt{x}} = 3 - \dfrac{6}{6} = 2$

(a) 1
(b) 2
(c) 3
(d) 4
(e) 6

6. The average cost (in dollars per unit) of producing q units is

$$\bar{c}(q) = 0.01q + 12 + \frac{900}{q}.$$

$C = \bar{C}q$

$C = 0.01q^2 + 12q + 900$

What is the marginal cost at $q = 20$?

$\dfrac{dC}{dq} = 0.02q + 12 \quad (q=20)$

$\dfrac{dC}{dq}(20) = 0.4 + 12$

(a) 11.6
(b) 12.4
(c) 12.5
(d) 13.0
(e) 12.5

7. If $p(q) = \dfrac{q+7}{q+2}$, find the marginal revenue $R'(q)$ at $q = 2$, where $R(q) = p(q)\,q$.

$R(q) = \dfrac{q+7}{q+2}\,q \qquad R'(q) = \dfrac{(q+2)(2q+7) - (q^2+7q)}{(q+2)^2}$

$R'(2) = \dfrac{4 \times 11 - 2 \times 9}{4^2} = \dfrac{26}{16}$

(a) 1
(b) 26/16
(c) 24/16
(d) 20/18
(e) 22/18

8. For $y = 5x^2 + 1$, the relative rate

(a) $\frac{50}{126}$

(b) $\frac{5}{13}$

(c) $\frac{10}{51}$

(d) $\frac{5}{12}$

(e) $\frac{10}{25}$

$\frac{y'}{y} = \quad 5x^2+1, \quad y|_{x=5} = \frac{10 \times 5}{5 \times 25 + 1} = \frac{50}{126}$

9. For $x^2 + y^2 = 25$, find $\frac{dy}{dx}$ at the point $(-4, 3)$.

(a) $\frac{3}{4}$

(b) $-\frac{3}{4}$

(c) $\frac{4}{3}$

(d) $-\frac{4}{3}$

(e) 0

$2x + 2y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{2x}{2y}$

$\left.\frac{dy}{dx}\right|_{(-4,3)} = -\frac{(-4)}{3} = 4/3$

10. Let $F(x) = (x + g(x))^3$ with $g(2) = -1$ and $g'(2) = 4$. Then $F'(2) =$

(a) 36

(b) 54

(c) 15

(d) 90

(e) 17

$F'(x) = 3(x+g(x)) \cdot (1+g'(x))$

$F'(2) = 3(2+g(2)) \cdot (1+g'(2))$

$= 3(2+(-1)) \cdot (1+4)$

$= 3 \cdot 5 = 15$