Now we turn to dq/dm. From the quotient and power rules, we have

$$\frac{dq}{dm} = \frac{d}{dm}\left(\frac{10m^2}{\sqrt{m^2+19}}\right)$$

$$= \frac{(m^2+19)^{1/2}\frac{d}{dm}(10m^2) - (10m^2)\frac{d}{dm}[(m^2+19)^{1/2}]}{[(m^2+19)^{1/2}]^2}$$

$$= \frac{(m^2+19)^{1/2}(20m) - (10m^2)[\frac{1}{2}(m^2+19)^{-1/2}(2m)]}{m^2+19}$$

so

$$\left.\frac{dq}{dm}\right|_{m=9} = \frac{(81+19)^{1/2}(20\cdot 9) - (10\cdot 81)[\frac{1}{2}(81+19)^{-1/2}(2\cdot 9)]}{81+19}$$

$$= 10.71$$

A direct formula for the marginal-revenue product is

$$\frac{dr}{dm} = \frac{dq}{dm}\left(p + q\frac{dp}{dq}\right)$$

Therefore, from the chain rule,

$$\left.\frac{dr}{dm}\right|_{m=9} = (1)(10.71) = 10.71$$

This means that if a tenth employee is hired, revenue will increase by approximately \$10.71 per day.

Now Work Problem 80 ◁

PROBLEMS 11.5

In Problems 1–8, use the chain rule.

1. If $y = u^2 - 2u$ and $u = x^2 - x$, find dy/dx.
2. If $y = 2u^3 - 8u$ and $u = 7x - x^3$, find dy/dx.
3. If $y = \dfrac{1}{w}$ and $w = 3x - 5$, find dy/dx.
4. If $y = \sqrt[4]{z}$ and $z = x^5 - x^4 + 3$, find dy/dx.
5. If $w = u^3$ and $u = \dfrac{t-1}{t+1}$, find dw/dt when $t = 1$.
6. If $z = u^2 + \sqrt{u} + 9$ and $u = 2s^2 - 1$, find dz/ds when $s = -1$.
7. If $y = 3w^2 - 8w + 4$ and $w = 2x^2 + 1$, find dy/dx when $x = 0$.
8. If $y = 2u^3 + 3u^2 + 5u - 1$ and $u = 3x + 1$, find dy/dx when $x = 1$.

In Problems 9–52, find y'.

9. $y = (3x+2)^6$
10. $y = (x^2-4)^4$
11. $y = (3+2x^3)^5$
12. $y = (x^2+x)^4$
13. $y = 5(x^3 - 3x^2 + 2x)^{100}$
14. $y = \dfrac{(2x^2+1)^4}{2}$
15. $y = (x^2-2)^{-3}$
16. $y = (2x^3 - 8x)^{-12}$
17. $y = 2(x^2+5x-2)^{-5/7}$
18. $y = 3(5x - 2x^3)^{-5/3}$
19. $y = \sqrt{5x^2 - x}$
20. $y = \sqrt{3x^2 - 7}$
21. $y = \sqrt[4]{2x-1}$
22. $y = \sqrt[3]{8x^2 - 1}$
23. $y = 4\sqrt[7]{(x^2+1)^3}$
24. $y = 7\sqrt[3]{(x^5-3)^5}$
25. $y = \dfrac{6}{2x^2 - x + 1}$
26. $y = \dfrac{3}{x^4+2}$
27. $y = \dfrac{1}{(x^2-3x)^2}$
28. $y = \dfrac{1}{(3+5x)^3}$
29. $y = \dfrac{4}{\sqrt{9x^2+1}}$
30. $y = \dfrac{3}{(3x^2-x)^{2/3}}$
31. $y = \sqrt[3]{7x} + \sqrt[3]{7x}$
32. $y = \sqrt{2x} + \dfrac{1}{\sqrt{2x}}$
33. $y = x^3(2x+3)^7$
34. $y = x(x+4)^4$
35. $y = 4x^2\sqrt{5x+1}$
36. $y = 4x^3\sqrt{1-x^2}$
37. $y = (x^2+2x-1)^3(5x)$
38. $y = x^4(x^4-1)^5$
39. $y = (8x-1)^3(2x+1)^4$
40. $y = (3x+2)^5(4x-5)^2$
41. $y = \left(\dfrac{x-3}{x+2}\right)^{12}$
42. $y = \left(\dfrac{2x}{x+2}\right)^4$
43. $y = \sqrt{\dfrac{x+1}{x-5}}$
44. $y = \sqrt[3]{\dfrac{8x^2-3}{x^2+2}}$
45. $y = \dfrac{2x-5}{(x^2+4)^3}$
46. $y = \dfrac{(4x-2)^4}{3x^2+7}$
47. $y = \dfrac{(8x-1)^5}{(3x-1)^3}$
48. $y = \sqrt[3]{(x-3)^3(x+5)}$
49. $y = 6(5x^2+2)\sqrt{x^4+5}$
50. $y = 6 + 3x - 4x(7x+1)^2$
51. $y = 8t + \dfrac{t-1}{t+4} - \left(\dfrac{8t-7}{4}\right)^2$
52. $y = \dfrac{(2x^3+6)(7x-5)}{(2x+4)^2}$

In Problems 53 and 54, use the quotient rule and power rule to find y'. Do not simplify your answer.

53. $y = \dfrac{(3x+2)^3(x+1)^4}{(x^2-7)^3}$
54. $y = \dfrac{\sqrt{x+2}(4x^2-1)^2}{9x-3}$
55. If $y = (5u+6)^3$ and $u = (x^2+1)^4$, find dy/dx when $x = 0$.
56. If $z = 2y^2 - 4y + 5$, $y = 6x - 5$, and $x = 2t$, find dz/dt when $t = 1$.
57. Find the slope of the curve $y = (x^2 - 7x - 8)^3$ at the point $(8, 0)$.

58. Find the slope of the curve $y = \sqrt{x+2}$ at the point $(7, 3)$.

In Problems 59–62, find an equation of the tangent line to the curve at the given point.

59. $y = \sqrt[3]{(x^2-8)^2}$; $(3, 1)$

60. $y = (x+3)^3$; $(-1, 8)$

61. $y = \dfrac{\sqrt{7x+2}}{x+1}$; $\left(1, \dfrac{3}{2}\right)$

62. $y = \dfrac{-3}{(3x^2+1)^3}$; $(0, -3)$

In Problems 63 and 64, determine the percentage rate of change of y with respect to x for the given value of x.

63. $y = (x^2+1)^4$; $x = 1$

64. $y = \dfrac{1}{(x^2-1)^3}$; $x = 2$

In Problems 65–68, q is the total number of units produced per day by m employees of a manufacturer, and p is the price per unit at which the q units are sold. In each case, find the marginal-revenue product for the given value of m.

65. $q = 5m, p = -0.4q + 50; m = 6$

66. $q = (200m - m^2)/20, p = -0.1q + 70; m = 40$

67. $q = 10m^2/\sqrt{m^2+9}, p = 525/(q+3); m = 4$

68. $q = 50m/\sqrt{m^2+11}, p = 100/(q+10); m = 5$

69. Demand Equation Suppose $p = 100 - \sqrt{q^2+20}$ is a demand equation for a manufacturer's product.
(a) Find the rate of change of p with respect to q.
(b) Find the relative rate of change of p with respect to q.
(c) Find the marginal-revenue function.

70. Marginal-Revenue Product If $p = k/q$, where k is a constant, is the demand equation for a manufacturer's product and $q = f(m)$ defines a function that gives the total number of units produced per day by m employees, show that the marginal-revenue product is always zero.

71. Cost Function The cost c of producing q units of a product is given by

$$c = 5500 + 12q + 0.2q^2$$

If the price per unit p is given by the equation

$$q = 900 - 1.5p$$

use the chain rule to find the rate of change of cost with respect to price per unit when $p = 85$.

72. Hospital Discharges A governmental health agency examined the records of a group of individuals who were hospitalized with a particular illness. It was found that the total proportion that had been discharged at the end of t days of hospitalization was given by

$$f(t) = 1 - \left(\frac{250}{250+t}\right)^3$$

Find $f'(100)$ and interpret your answer.

73. Marginal Cost If the total-cost function for a manufacturer is given by

$$c = \frac{4q^2}{\sqrt{q^2+2}} + 6000$$

find the marginal-cost function.

74. Salary/Education For a certain population, if E is the number of years of a person's education and S represents average annual salary in dollars, then for $E \geq 7$,

$$S = 340E^2 - 4360E + 42{,}800$$

(a) How fast is salary changing with respect to education when $E = 16$?
(b) At what level of education does the rate of change of salary equal \$5000 per year of education?

75. Biology The volume of a spherical cell is given by $V = \frac{4}{3}\pi r^3$, where r is the radius. At time t seconds, the radius (in centimeters) is given by

$$r = 10^{-8}t^2 + 10^{-7}t$$

Use the chain rule to find dV/dt when $t = 10$.

76. Pressure in Body Tissue Under certain conditions, the pressure p developed in body tissue by ultrasonic beams is given as a function of the beam's intensity via the equation[15]

$$p = (2\rho VI)^{1/2}$$

where ρ (a Greek letter read "rho") is density of the affected tissue and V is the velocity of propagation of the beam. Here ρ and V are constants. (a) Find the rate of change of p with respect to I. (b) Find the relative rate of change of p with respect to I.

77. Demography Suppose that, for a certain group of 20,000 births, the number of people surviving to age x years is

$$l_x = -0.000354x^4 + 0.00452x^3 + 0.848x^2 - 34.9x + 20{,}000$$
$$0 \leq x \leq 95.2$$

(a) Find the rate of change of l_x with respect to x, and evaluate your answer for $x = 65$.
(b) Find the relative rate of change and the percentage rate of change of l_x when $x = 65$. Round your answers to three decimal places.

78. Muscle Contraction A muscle has the ability to shorten when a load, such as a weight, is imposed on it. The equation

$$(P + a)(v + b) = k$$

is called the "fundamental equation of muscle contraction."[16] Here P is the load imposed on the muscle, v is the velocity of the shortening of the muscle fibers, and a, b, and k are positive constants. Express v as a function of P. Use your result to find dv/dP.

79. Economics Suppose $pq = 100$ is the demand equation for a manufacturer's product. Let c be the total cost, and assume that the marginal cost is 0.01 when $q = 200$. Use the chain rule to find dc/dp when $q = 200$.

80. Marginal-Revenue Product A monopolist who employs m workers finds that they produce

$$q = 2m(2m+1)^{3/2}$$

units of product per day. The total revenue r (in dollars) is given by

$$r = \frac{50q}{\sqrt{1000+3q}}$$

[15] R. W. Stacy et al., *Essentials of Biological and Medical Physics* (New York: McGraw-Hill Book Company, 1955).
[16] Ibid.