PROBLEMS 11.4

In Problems 1–48, differentiate the functions.

1. $f(x) = (4x + 1)(6x + 3)$
2. $f(x) = (3x - 1)(7x + 2)$
3. $s(t) = (5 - 3t)(t^3 - 2t^2)$
4. $Q(x) = (x^2 + 3x)(7x^2 - 5)$
5. $f(r) = (3r^2 - 4)(r^2 - 5r + 1)$
6. $C(I) = (2I^2 - 3)(3I^2 - 4I + 1)$
7. $f(x) = x^2(2x^2 - 5)$
8. $f(x) = 3x^3(x^2 - 2x + 2)$
9. $y = (x^2 + 5x - 7)(6x^2 - 5x + 4)$
10. $\phi(x) = (3 - 5x + 2x^2)(2 + x - 4x^2)$
11. $f(w) = (w^2 + 3w - 7)(2w^3 - 4)$
12. $f(x) = (3x - x^2)(3 - x - x^2)$
13. $y = (x^2 - 1)(3x^3 - 6x + 5) - 4(4x^2 + 2x + 1)$
14. $h(x) = 5(x^7 + 4) + 4(5x^3 - 2)(4x^2 + 7x)$
15. $F(p) = \frac{3}{2}(5\sqrt{p} - 2)(3p - 1)$
16. $g(x) = (\sqrt{x} + 5x - 2)(\sqrt[3]{x} - 3\sqrt{x})$
17. $y = 7 \cdot \frac{2}{3}$
18. $y = (x - 1)(x - 2)(x - 3)$
19. $y = (5x + 3)(2x - 5)(7x + 9)$
20. $y = \dfrac{2x - 3}{4x + 1}$
21. $f(x) = \dfrac{5x}{x - 1}$
22. $H(x) = \dfrac{-5x}{5 - x}$
23. $f(x) = \dfrac{-13}{3x^5}$
24. $f(x) = \dfrac{3(5x^2 - 7)}{4}$
25. $y = \dfrac{x + 2}{x - 1}$
26. $h(w) = \dfrac{3w^2 + 5w - 1}{w - 3}$
27. $h(z) = \dfrac{6 - 2z}{z^2 - 4}$
28. $z = \dfrac{2x^2 + 5x - 2}{3x^2 + 5x + 3}$
29. $y = \dfrac{4x^2 + 3x + 2}{3x^2 - 2x + 1}$
30. $f(x) = \dfrac{x^3 - x^2 + 1}{x^2 + 1}$
31. $y = \dfrac{x^2 - 4x + 3}{2x^2 - 3x + 2}$
32. $F(z) = \dfrac{z^4 + 4}{3z}$
33. $g(x) = \dfrac{1}{x^{100} + 7}$
34. $y = \dfrac{-8}{7x^6}$
35. $u(v) = \dfrac{v^3 - 8}{v}$
36. $y = \dfrac{x - 5}{8\sqrt{x}}$
37. $y = \dfrac{3x^2 - x - 1}{\sqrt[3]{x}}$
38. $y = \dfrac{x^{0.3} - 2}{2x^{2.1} + 1}$
39. $y = 1 - \dfrac{5}{2x + 5} + \dfrac{2x}{3x + 1}$
40. $q(x) = 2x^3 + \dfrac{5x + 1}{3x - 5} - \dfrac{2}{x^3}$
41. $y = \dfrac{x - 5}{(x + 2)(x - 4)}$
42. $y = \dfrac{(9x - 1)(3x + 2)}{4 - 5x}$
43. $s(t) = \dfrac{t^2 + 3t}{(t^2 - 1)(t^3 + 7)}$
44. $f(s) = \dfrac{17}{s(4s^3 + 5s - 23)}$
45. $y = 3x - \dfrac{\frac{2}{x} - \frac{3}{x - 1}}{x - 2}$
46. $y = 3 - 12x^3 + \dfrac{1 - \frac{5}{x^2 + 2}}{x^2 + 5}$
47. $f(x) = \dfrac{a + x}{a - x}$, where a is a constant
48. $f(x) = \dfrac{x^{-1} + a^{-1}}{x^{-1} - a^{-1}}$, where a is a constant
49. Find the slope of the curve $y = (2x^2 - x + 3)(x^3 + x + 1)$ at $(1, 12)$.
50. Find the slope of the curve $y = \dfrac{x^3}{x^4 + 1}$ at $(-1, -\frac{1}{2})$.

In Problems 51–54, find an equation of the tangent line to the curve at the given point.

51. $y = \dfrac{6}{x - 1}$; $(3, 3)$
52. $y = \dfrac{x + 5}{x^2}$; $(1, 6)$
53. $y = (2x + 3)[2(x^4 - 5x^2 + 4)]$; $(0, 24)$
54. $y = \dfrac{x - 1}{x(x^2 + 1)}$; $(2, \frac{1}{10})$

In Problems 55 and 56, determine the relative rate of change of y with respect to x for the given value of x.

55. $y = \dfrac{x}{2x - 6}$; $x = 1$
56. $y = \dfrac{1 - x}{1 + x}$; $x = 5$

57. Motion The position function for an object moving in a straight line is

$$s = \frac{2}{t^3 + 1}$$

where t is in seconds and s is in meters. Find the position and velocity of the object at $t = 1$.

58. Motion The position function for an object moving in a straight-line path is

$$s = \frac{t + 3}{t^2 + 7}$$

where t is in seconds and s is in meters. Find the positive value(s) of t for which the velocity of the object is 0.

In Problems 59–62, each equation represents a demand function for a certain product, where p denotes the price per unit for q units. Find the marginal-revenue function in each case. Recall that revenue = pq.

59. $p = 80 - 0.02q$
60. $p = 500/q$
61. $p = \dfrac{108}{q + 2} - 3$
62. $p = \dfrac{q + 750}{q + 50}$

63. Consumption Function For the United States (1922–1942), the consumption function is estimated by[11]

$$C = 0.672I + 113.1$$

Find the marginal propensity to consume.

64. Consumption Function Repeat Problem 63 for $C = 0.836I + 127.2$.

In Problems 65–68, each equation represents a consumption function. Find the marginal propensity to consume and the marginal propensity to save for the given value of I.

65. $C = 3 + \sqrt{I} + 2\sqrt[3]{I}$; $I = 1$
66. $C = 6 + \dfrac{3I}{4} - \dfrac{\sqrt{I}}{3}$; $I = 25$

[11] T. Haavelmo, "Methods of Measuring the Marginal Propensity to Consume," *Journal of the American Statistical Association*, XLII (1947), 105–22.