Each question worth 10 points

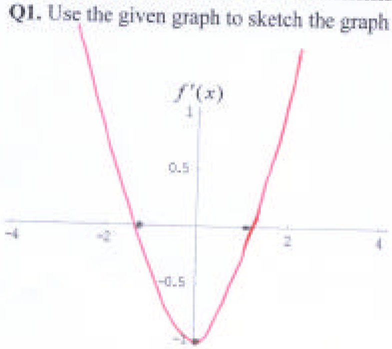
King Fuhd University of Petroleum and Minerals
Department of Mathematical Sciences

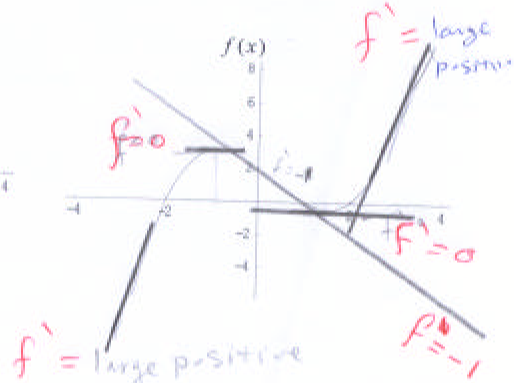
Math 101
Second Exam , Semester 022
Time:10:00-11:00pm, April 30, 2003

100

Name :--ID # :------------------------Section #----

Q1. Use the given graph to sketch the graph of the derivative.





Q2. Find y'' where $yx = \sec(y^2+1)$

$$y'x + y = \sec(y^2+1)\tan(y^2+1)\cdot(2yy')$$

Then
$$y' = \frac{y}{2y\sec(y^2+1)\tan(y^2+1) - x}$$

Now
$$y'' = \frac{\{y'(2y\sec(y^2+1)\tan(y^2+1) - x\} - \{y\left(\begin{array}{l}2y'\sec(y^2+1)\tan(y^2+1)\\ +2y\sec(y^2+1)\tan^2(y^2+1)2yy'\\ +2y\sec^3(y^2+1)2yy' - 1\end{array}\right)}{\left[2y\sec(y^2+1)\tan(y^2+1) - x\right]^2}$$

Q3. If $\pi^\pi = y\sec x$ prove that $y' = -\pi^\pi \sin x$

$$y = \frac{\pi^\pi}{\sec x}$$
$$y = \pi^\pi \cos x$$
$$y' = -\pi^\pi \sin x$$

$$0 = y'\sec x + y\sec x\tan x$$

then
$$y' = \frac{-y\cancel{\sec x}\tan x}{\cancel{\sec x}} = -\frac{\pi^\pi}{\sec x}\cdot\tan x$$

$$= -\pi^\pi \cancel{\cos x}\cdot\frac{\sin x}{\cancel{\cos x}}$$

$$= -\pi^\pi \sin x.$$

Dr Al-Homidan HomePage

Q4. Use the definition of derivative to find $f'(x)$ where $f(x) = \frac{3}{\sqrt{1-x}}$

$$\lim_{h\to 0} \frac{\frac{3}{\sqrt{1-(x+h)}} - \frac{3}{\sqrt{1-x}}}{h} = \lim_{h\to 0} \frac{\frac{3\left(\sqrt{1-x}-\sqrt{1-(x+h)}\right)}{\sqrt{(1-(x+h))(1-x)}} \cdot \frac{\left(\sqrt{1-x}+\sqrt{1-(x+h)}\right)}{\left(\sqrt{1-x}+\sqrt{1-(x+h)}\right)}}{h}$$

$$= \lim_{h\to 0} \frac{3\left(\cancel{1-x}-\cancel{1-x}+h\right)}{h\left\{\sqrt{(1-x+h)(1-x)}\left[\sqrt{1-x}+\sqrt{1-(x+h)}\right]\right\}} = \frac{-3}{\sqrt{(1-x)^2}\left(2\sqrt{1-x}\right)} = \frac{-3}{2(1-x)^{3/2}}$$

Q5. If $f(x)$ and $h(x)$ are differentiable function at $x = 2$, such that $f(2) = 3, f'(2) = -1$, $h(2) = 2, h'(2) = 1$, and $g(x) = xf(h(x)) + 2f(x)/h(x)$, then find $g'(2)$

$$g'(x) = f(h(x)) + x f'(h(x))\, h'(x) + 2\frac{f'(x)h(x) - f(x)h'(x)}{h^2(x)}$$

$$g'(2) = f(h(2)) + 2f'(h(2))\boxed{h'(2)} + 2\frac{f'(2)h(2) - f(2)h'(2)}{h^2(2)}$$

$$= f(2) + 2f'(2)\,1 + 2\frac{(-1)(2) - 3(1)}{4}$$

$$= 3 + 2(-1) + 2\left(\frac{-5}{4}\right) = -3/2$$

Q6. Show that the curve $y = 6x^3 + 5x - 3$ has no tangent line with slope 4.

$$y' = 18x^2 + 5 = 4$$

$$18x^2 = -1$$

$$x = \pm\sqrt{-\frac{1}{18}}$$

complex zeros

no slope

Q7. Find all equations of the tangent lines to the parable $y = x^2 + x$ that passes through the point (2,-3).

$y' = 2x+1$ at the point (x_0, y_0) then $y' = 2x_0+1$

The equation of the line with slope $2x_0+1$ that passes through $(2,-3)$ is $y_0 + 3 = (2x_0+1)(x_0-2)$

But (x_0, y_0) on the curve So $y_0 = x_0^2 + x_0$ then

$x_0^2 + x_0 + 3 = (2x_0+1)(x_0-2) \Rightarrow x_0^2 - 4x_0 - 5 = 0$

$(x_0+1)(x_0-5) = 0 \quad x_0 = -1$ or $x_0 = 5$

Eq① $\Rightarrow y + 3 = -(x-2)$

Eq② $\Rightarrow y + 3 = 11(x-2)$.

Q8. Prove that every differentiable function is continuous function.

f is differentiable means $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ exist.

or $= \lim_{x \to w} \frac{f(x) - f(w)}{x - w}$

we need to prove that

$\boxed{\lim_{x \to a} f(x) = f(a)}$

Now $f(x) = f(x) - f(a) + f(a)$

$= \frac{f(x) - f(a)}{x - a} \cdot x - a + f(a)$

Then $\lim_{x \to a} f(x) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} \cdot \lim_{x \to a} (x - a) + \lim_{x \to a} f(a)$

$= f'(a) \cdot 0 + f(a)$

Q9. If $f(x)=\dfrac{x+1}{x-1}$ find $f^{-1}(x)$ then find the domain and range of $f^{-1}(x)$

$y=\dfrac{x+1}{x-1} \qquad (x-1)y = x+1 \Rightarrow xy-y=x+1$

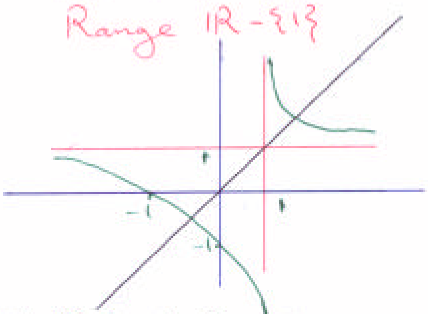
$\Rightarrow xy-x = 1+y$

$x(y-1)=y+1$

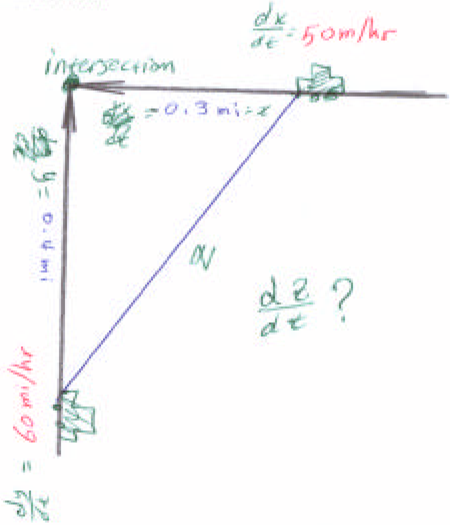
$\Rightarrow f^{-1}(x)=\dfrac{x+1}{x-1}$

Domain $\mathbb{R}-\{1\}$

Range $\mathbb{R}-\{1\}$



Q10. Car A is traveling west at 50 m/hr and car B is traveling north at 60 m/hr. Both are headed for the intersection of the two roads. At what rate are the cars approaching each other when car A is 0.3 mi and car B is 0.4 mi from the intersection.



$x^2+y^2=z^2$

$\cancel{2}x\dfrac{dx}{dt}+\cancel{2}y\dfrac{dy}{dt}=\cancel{2}z\dfrac{dz}{dt}$

First find $z=0.5$ mi $\sqrt{(0.3)^2+(0.4)^2}$

then

$\dfrac{dz}{dt}=\dfrac{(0.3)(50)+(0.4)(60)}{0.5}$

$= 78$ mi/hr.