Q1. $1+0=\frac{1}{1+(\pi-y)^2}\cdot -y' \Rightarrow y'=-1-(\pi-y)^2$

$y''=0-2(\pi-y)\cdot -y'=2y'(\pi-y)$

Q2 $\frac{dy}{dx}=\frac{dy}{dt}\cdot\frac{dt}{dx}$ $\quad \frac{1}{\sqrt{1-t^2}}=y+t\frac{dy}{dt} \Rightarrow \frac{dy}{dt}=\left[\frac{1}{\sqrt{1-t^2}}-y\right]/t$

$\frac{dy}{dt}=\frac{1-y\sqrt{1-t^2}}{t\sqrt{1-t^2}} \Rightarrow \frac{dy}{dx}=\frac{dy}{dt}\cdot\frac{dt}{dx}=\frac{1-y\sqrt{1-t^2}}{t\sqrt{1-t^2}}\cdot\frac{1}{\sqrt{1-t^2}}=\frac{1-y\sqrt{1-t^2}}{t(1-t^2)}$

Q3 $f(x)=\sqrt{x}$ $\quad f(48)\simeq f(x_0)+f'(x_0)(x-x_0)=\sqrt{49}+\frac{1}{2\sqrt{49}}(-1)$

$=7-\frac{1}{14}$

Q4 I. $y'=2x$ $\quad y''=2$ $\quad y^{(n)}=0$ $\quad n>2$ $\quad$ II. $y'=2e^{2x}$ $\quad y''=2^2e^{2x}$

$y^{(n)}=2^n e^{2x}$

Q5 $y=e^{\frac{x+1}{x-1}}$ $\quad \ln y=\frac{x+1}{x-1}$ $\quad (x-1)\ln y=x+1$

$x\ln y-x=+\ln y+1 \Rightarrow x(-1+\ln y)=1+\ln y$

$x=\frac{1+\ln y}{-1+\ln y}$ $\quad f^{-1}(x)=\frac{1+\ln x}{-1+\ln x}$ $\quad$ $\ln x$ define if $x\in(0,\infty)$

but if $\ln x=1$ denominator $=0$

$\Rightarrow \underline{x=e}$

The Domain $(0,e)\cup(e,\infty)$

Q6 $y'=\frac{1}{7}(2+\tan x^2)^{-\frac{6}{7}}\cdot\sec^2 x^2\cdot 2x+\frac{1}{3x\ln 10}\cdot 3$

Q7 $(\infty-\infty)$ $\quad \cot^2 x-\csc^2 x=-1 \Rightarrow \lim_{x\to 0}-1=-1$

Q8. (1^∞) $\quad \lim_{x\to 0^+}\csc x\ln(1+3x)$ $\quad (\infty\cdot 0)=\lim_{x\to 0^+}\frac{\ln(1+3x)}{\sin x}$ $\left(\frac{0}{0}\right)$

L'Hopital $\lim_{x\to 0^+}\frac{\frac{1}{1+3x}\cdot 3}{\cos x}=3.$ $\quad \Rightarrow \lim_{x\to 0^+}f(x)=e^3$

Q9

ⓐ If $y=2 \Rightarrow 4x(8)=8(4) \Rightarrow x=1$, derivative w.r.t. t. $4\frac{dx}{dt}y^3+x\,3y^2\frac{dy}{dt}=16\frac{dy}{dt}$

$\frac{dy}{dt}=-12$ unit/second. ⓑ Falling

Q10 $x=1$ $\quad y^2+y-2=0$ $\quad (y-1)(y+2)=0$ $\quad y=1$ or $y=-2$ $\quad (1,1)$ $(1,-2)$

$y^2+2xyy'+y+xy'=0$

$y'=\frac{-y^2-y}{2xy+x}$ $\quad y'\Big|_{(1,1)}=\frac{-2}{3}$ $\quad (y-1)=-\frac{2}{3}(x-1)$ ← Eq#1

$y'\Big|_{(1,-2)}=\frac{-2}{-3}=\frac{2}{3}$ $\quad y+2=\frac{2}{3}(x-1)$ ← Eq.#2