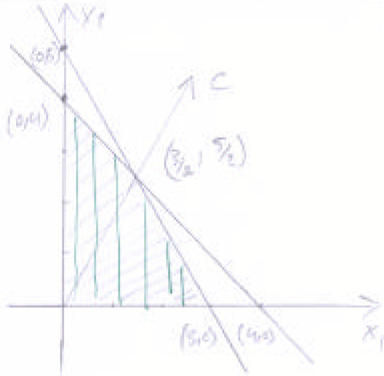
1 ① max $Z = 120x_1 + 100x_2$

st $x_1 + x_2 \le 4$

$5x_1 + 3x_2 \le 15$

$x_1, x_2 \ge 0$



Add slack variables:

$x_1 + x_2 + x_3 = 4$

$5x_1 + 3x_2 + x_4 = 15$

$A = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 5 & 3 & 0 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} 4 \\ 15 \end{bmatrix}$

number of possible solutions $= \dfrac{n!}{m!\,(n-m)!} = \dfrac{4!}{2!\,2!} = \boxed{6}$

① $B = \begin{bmatrix} 1 & 1 \\ 5 & 3 \end{bmatrix} \Rightarrow B^{-1}b = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3/2 \\ 5/2 \end{bmatrix} \Rightarrow x = \begin{bmatrix} 3/2 \\ 5/2 \\ 0 \\ 0 \end{bmatrix}$

② $B = \begin{bmatrix} 1 & 1 \\ 5 & 0 \end{bmatrix} \Rightarrow B^{-1}b = \begin{bmatrix} x_1 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix} \Rightarrow x = \begin{bmatrix} 3 \\ 0 \\ 1 \\ 0 \end{bmatrix}$

③ $B = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix} \Rightarrow B^{-1}b = \begin{bmatrix} x_1 \\ x_4 \end{bmatrix} = \begin{bmatrix} 4 \\ -5 \end{bmatrix} \Rightarrow x = \begin{bmatrix} 4 \\ 0 \\ 0 \\ -5 \end{bmatrix}$ Not Feasible

④ $B = \begin{bmatrix} 1 & 1 \\ 3 & 0 \end{bmatrix} \Rightarrow B^{-1}b = \begin{bmatrix} x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 \\ -1 \end{bmatrix} \Rightarrow x = \begin{bmatrix} 0 \\ 5 \\ -1 \\ 0 \end{bmatrix}$ Not Feasible

⑤ $B = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} \Rightarrow B^{-1}b = \begin{bmatrix} x_2 \\ x_4 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix} \Rightarrow x = \begin{bmatrix} 0 \\ 4 \\ 0 \\ 3 \end{bmatrix}$

⑥ $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow B^{-1}b = \begin{bmatrix} x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 4 \\ 15 \end{bmatrix} \Rightarrow x = \begin{bmatrix} 0 \\ 0 \\ 4 \\ 15 \end{bmatrix}$

The Basic Feasible Solutions

$(3/2, 5/2)$, $(3, 0)$, $(0, 4)$, $(0, 0)$

11 (a)

Finding the optimal solutions:

$z = 120(3/2) + 100(5/2) = 430$

$z = 120(3) + 0 = 360$

$z = 120(0) + 100(4) = 400$

$z = 0 + 0 = 0$

The optimal BFS is $(3/2, 5/2)$.

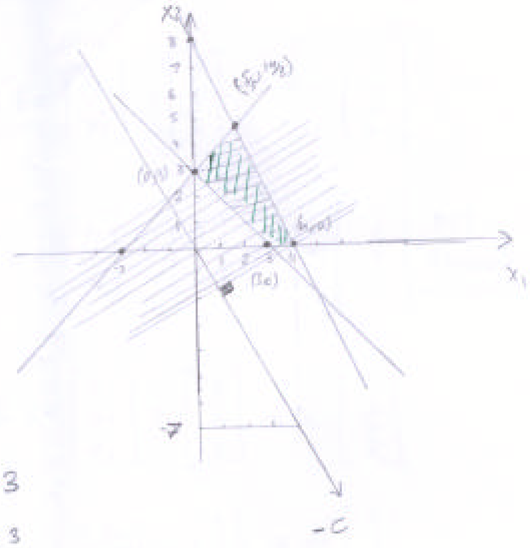
(a) min $z = -4x_1 + 7x_2$

sb $x_1 + x_2 \geq 3$

$-x_1 + x_2 \leq 3$

$2x_1 + x_2 \leq 8$

$x_1, x_2 \geq 0$



Adding Slacking Variables:

$$
\begin{aligned}
x_1 + x_2 - x_3 &= 3 \\
-x_1 + x_2 + x_4 &= 3 \\
2x_1 + x_2 + x_5 &= 8
\end{aligned}
$$

$$A = \begin{bmatrix} 1 & 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 1 & 0 \\ 2 & 1 & 0 & 0 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} 3 \\ 3 \\ 8 \end{bmatrix}$$

number of possible solutions $= \dfrac{n!}{m!(n-m)!} = \dfrac{5!}{3!(2!)} = \boxed{10}$

① $B = \begin{bmatrix} 1 & 1 & -1 \\ -1 & 1 & 0 \\ 2 & 1 & 0 \end{bmatrix} \Rightarrow B^{-1}b = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5/3 \\ 14/3 \\ 10/3 \end{bmatrix} \Rightarrow x = \begin{bmatrix} 5/3 \\ 14/3 \\ 10/3 \\ 0 \\ 0 \end{bmatrix}$

② $B = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 1 & 1 \\ 2 & 1 & 0 \end{bmatrix} \Rightarrow B^{-1}b = \begin{bmatrix} x_1 \\ x_2 \\ x_4 \end{bmatrix} = \begin{bmatrix} 5 \\ -2 \\ 10 \end{bmatrix}$ not Feasible

③ $B = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 1 & 0 \\ 2 & 1 & 1 \end{bmatrix} \Rightarrow B^{-1}b = \begin{bmatrix} x_1 \\ x_2 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 5 \end{bmatrix} \Rightarrow x = \begin{bmatrix} 0 \\ 3 \\ 0 \\ 0 \\ 5 \end{bmatrix}$

④ $B = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 0 & 1 \\ 2 & 0 & 0 \end{bmatrix} \Rightarrow B^{-1}b = \begin{bmatrix} x_1 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 7 \end{bmatrix} \Rightarrow x = \begin{bmatrix} 4 \\ 0 \\ 1 \\ 7 \\ 0 \end{bmatrix}$

⑤ $B = \begin{bmatrix} +1 & -1 & 0 \\ -1 & 0 & 0 \\ 2 & 0 & 1 \end{bmatrix} \Rightarrow B^{-1}b = \begin{bmatrix} x_1 \\ x_3 \\ x_5 \end{bmatrix} = \begin{bmatrix} -3 \\ -6 \\ 14 \end{bmatrix} \Rightarrow x = \begin{bmatrix} -3 \\ 0 \\ -6 \\ 0 \\ 14 \end{bmatrix}$ Not Feasible

(6) $B = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} \Rightarrow B^{-1}b = \begin{bmatrix} x_1 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \\ 2 \end{bmatrix} \Rightarrow x = \begin{bmatrix} 3 \\ 0 \\ 0 \\ 6 \\ 2 \end{bmatrix}$

(7) $B = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \Rightarrow B^{-1}b = \begin{bmatrix} x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 8 \\ 5 \\ -5 \end{bmatrix} \Rightarrow x = \begin{bmatrix} 0 \\ 8 \\ 5 \\ -5 \\ 0 \end{bmatrix}$ not Feasible

(8) $B = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix} \Rightarrow B^{-1}b = \begin{bmatrix} x_2 \\ x_3 \\ x_5 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 5 \end{bmatrix} \Rightarrow x = \begin{bmatrix} 0 \\ 3 \\ 0 \\ 0 \\ 5 \end{bmatrix}$

(9) $B = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \Rightarrow B^{-1}b = \begin{bmatrix} x_2 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 5 \end{bmatrix} \Rightarrow x = \begin{bmatrix} 0 \\ 3 \\ 0 \\ 0 \\ 5 \end{bmatrix}$

(10) $B = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow B^{-1}b = \begin{bmatrix} x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} -3 \\ 3 \\ 6 \end{bmatrix}$ Not Feasible

The Basic Feasible solutions are

$(5/3, 14/3)$, $(4,0)$, $(3,0)$, $(0,3)$

Finding the optimal solution:

(1) $z = -4(5/3) + 7(14/3) = 26$

(2) $z = -4(4) + 0 = -16$ ←

(3) $z = -4(3) + 6 = -12$

(4) $z = 0 + 7(3) = 21$

The Optimal Basic Feasible solution is $\boxed{\begin{matrix} x_1 = 4 \\ x_2 = 0 \end{matrix}}$

3) Max $Z = 2x_1 + 2x_2$

s.t. $x_1 + 2x_2 \leq 4$

$x_1 + x_2 \leq 3$

$x_1 - x_2 \geq 1$

$x_1 \geq 0 \,,\; x_2 \geq 0$

Adding slack variable

$$\begin{aligned} x_1 + 2x_2 + x_3 &= 4 \\ x_1 + x_2 + x_4 &= 3 \\ x_1 - x_2 - x_5 &= 1 \end{aligned}$$

$$A = \begin{bmatrix} 1 & 2 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 & -1 \end{bmatrix} \,,\quad b = \begin{bmatrix} 4 \\ 3 \\ 1 \end{bmatrix} \,,\quad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}$$

number of possible solutions $= \dfrac{5!}{3!\,2!} = 10$

① $B = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 0 \\ 1 & -1 & 0 \end{bmatrix} \Rightarrow B^{-1}b = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \Rightarrow X = \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ Degenerate BFS

② $B = \begin{bmatrix} 1 & 2 & 0 \\ 1 & 1 & 1 \\ 1 & -1 & 0 \end{bmatrix} \Rightarrow B^{-1}b = \begin{bmatrix} x_1 \\ x_2 \\ x_4 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \Rightarrow X = \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ Degenerate BFS

③ $B = \begin{bmatrix} 1 & 2 & 0 \\ 1 & 1 & 0 \\ 1 & -1 & -1 \end{bmatrix} \Rightarrow B^{-1}b = \begin{bmatrix} x_1 \\ x_2 \\ x_5 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \Rightarrow X = \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ Degenerate BFS

④ $B = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \Rightarrow B^{-1}b = \begin{bmatrix} x_1 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} \Rightarrow X = \begin{bmatrix} 1 \\ 0 \\ 3 \\ 2 \\ 0 \end{bmatrix}$

⑤ $B = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & -1 \end{bmatrix} \Rightarrow B^{-1}b = \begin{bmatrix} x_1 \\ x_3 \\ x_5 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix} \Rightarrow X = \begin{bmatrix} 3 \\ 0 \\ 1 \\ 0 \\ 2 \end{bmatrix}$

(6) $B = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix} \Rightarrow B^{-1}b = \begin{bmatrix} x_1 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 4 \\ -1 \\ 3 \end{bmatrix}$ Not Feasible

(7) $B = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & 1 \\ -1 & 0 & 0 \end{bmatrix} \Rightarrow B^{-1}b = \begin{bmatrix} x_2 \\ x_1 \\ x_4 \end{bmatrix} = \begin{bmatrix} -1 \\ 6 \\ 4 \end{bmatrix}$ Not Feasible

(8) $B = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & 0 \\ -1 & 0 & -1 \end{bmatrix} \Rightarrow B^{-1}b = \begin{bmatrix} x_2 \\ x_3 \\ x_5 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -4 \end{bmatrix}$ Not Feasible

(9) $B = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 1 & 0 \\ -1 & 0 & -1 \end{bmatrix} \Rightarrow B^{-1}b = \begin{bmatrix} x_2 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix}$ Not Feasible

(10) $B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \Rightarrow B^{-1}b = \begin{bmatrix} x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ -1 \end{bmatrix}$ Not Feasible

The Basic Feasible solutions are

$(3,0)$, $(1,0)$

The Degenerate BFS is $(2,1)$

Finding the Optimal solution:

$Z = 2(3) + 2(0) = 6$

$Z = 2(1) + 2(0) = 2$

$Z = 2(2) + 2(1) = 6$

The optimal solutions are $(3,0)$ and $(2,1)$ and all the points between them (infinite number of solutions)