(1) Given $f(x) = \frac{x^4}{12} - x^2$, find the second derivative

Q4

$f'(x) = \frac{x^3}{3} - 2x \Rightarrow f''(x) = x^2 - 2$. (1½)

Set $f''(x) = 0$: $x^2 - 2 = 0 \Rightarrow x = \pm\sqrt{2}$

Therefore, Concave up: $f''(x) > 0 \Rightarrow x^2 > 2$ (½)

$(-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$

Concave down: $f''(x) < 0 \Rightarrow x^2 < 2$

$(-\sqrt{2}, \sqrt{2})$ (1)

To find the inflection points are

$f(\pm\sqrt{2}) = \frac{(\sqrt{2})^4}{12} - (\sqrt{2})^2 = \frac{4}{12} - 2 = -\frac{5}{3}$ (½)

Hence $(-\sqrt{2}, -\frac{5}{3})$ and $(\sqrt{2}, -\frac{5}{3})$

(2) Given $f(x) = x^4 - 2x^3$, we start by finding the critical points

$f'(x) = 4x^3 - 6x^2 = 2x^2(2x-3)$.

Set $f'(x) = 0$: $x = 0$ or $x = \frac{3}{2}$ (1)

$f''(x) = 12x^2 - 12x = 12x(x-1)$.

At $x = \frac{3}{2}$: $f''(\frac{3}{2}) = 12(\frac{3}{2})(\frac{1}{2}) = 9 > 0$. (1)

Therefore, $x = \frac{3}{2}$ is a local minimum.

$f(\frac{3}{2}) = (\frac{3}{2})^4 - 2(\frac{3}{2})^3 = \frac{81}{16} - \frac{108}{16} = -\frac{27}{16}$.

Hence, the minimum is $(\frac{3}{2}, -\frac{27}{16})$ (1)

At $x = 0$: $f''(0) = 0$. Therefore, we have a saddle point $(0,0)$. (½)

Q 4

(3) Given $f(x) = \dfrac{10x^2+9x+5}{5x+2}$

$$5x+2 \;\overline{)\; 10x^2+9x+5}$$

$2x+1$

$10x^2+4x$ (1/2)

$5x+5$

$5x+2$

3.

$\therefore \quad f(x) = 2x+1 + \dfrac{3}{5x+2}$.

The oblique asymptotes is $f(x) = 2x+1$. (1)

The denominator $5x+2=0 \Rightarrow x = -2/5$ //

The vertical asymptotes is at $x = -2/5$ (1)

$$\lim_{x\to\infty} \frac{10x^2+9x+5}{5x+2} = \frac{\frac{10x^2}{x^2}+\frac{9x}{x^2}+\frac{5}{x^2}}{\frac{5x}{x^2}+\frac{2}{x^2}} = \lim_{x\to\infty} \frac{10+\frac{9}{x}+\frac{5}{x^2}}{\frac{5}{x}+\frac{2}{x^2}}$$

$$= \frac{10+0+0}{0+0} = \infty.$$ (1)

There is no H.A here //