Marking scheme Math 106 semester 261 Quiz 3 #111

Q3

Q1. Marginal cost: Given the cost function

$$C(q) = q^{\sqrt{q+1}}$$

Then, $\ln C = \sqrt{q+1}\ln q$ (1)

Differentiating (1)

$$\frac{C'(q)}{C(q)} = \ln q\left(\tfrac{1}{2}(q+1)^{-\frac{1}{2}}\right) + \sqrt{q+1}\left(\tfrac{1}{q}\right) \quad (1/2)\ (1/2)$$

$$= \frac{\ln q}{2\sqrt{q+1}} + \frac{\sqrt{q+1}}{q} \quad (1/2)\ (1/2)$$

Therefore, the marginal cost is

$$C'(q) = q^{\sqrt{q+1}}\left(\frac{\ln q}{2\sqrt{q+1}} + \frac{\sqrt{q+1}}{q}\right)$$

for $q > 0$.

or $$MC = q^{\sqrt{q+1}}\left[\frac{\ln q}{2\sqrt{q+1}} + \frac{\sqrt{q+1}}{q}\right] \quad (1)$$

Q2. Given that $f(x) = x^{2/3}$ with $x \in [-8, 8]$.

we first evaluate the endpoints and the critical value $x = 0$ (1)

i.e., $f(-8) = (-8)^{2/3} = (-2^3)^{2/3} = (-2)^2 = 4$ (1)

$f(0) = (0)^{2/3} = 0$ (1)

$f(8) = (8)^{2/3} = (2^3)^{2/3} = 2^2 = 4$ (1)

Thus, $M = 4$, and $m = 0$. Therefore, $M \times m = 4(0) = 0$ (1)

(1)