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List of Symbols

$\mathbf{u}$	fluid velocity
ϕ	porosity
ϕ_0	initial porosity
ϕ_f	final porosity
$\kappa(\phi)$	porosity-dependent permeability function
p	fluid pressure
c	concentration of the soluble component
c_{eq}	equilibrium concentration
ρ	molar density of the mineral being dissolved
k	reaction rate constant
$\mathbf{F}$	advective flux
$\mathbf{G}$	Godunov approximation to $\mathbf{F}$
K	generic positive constant independent of h
ν	unit outward normal
Ω	domain; usually open, bounded, simply connected subset
$\partial\Omega$	boundary of Ω
RT0	lowest order Raviart-Thomas spaces
Δt	time step