1)

$$f(x) = \begin{cases} K(1-x^2) & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

i) $$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_0^1 K(1-x^2)\,dx = 1$$

$$K\left[x - \frac{x^3}{3}\right]_0^1 = 1 \Rightarrow K = \frac{3}{2}$$

ii) $$P(0.2 \le X \le 0.4) = \int_{0.2}^{0.4} \frac{3}{2}(1-x^2)\,dx$$

$$= \frac{3}{2}\left[X - \frac{X^3}{3}\right]_{0.2}^{0.4}$$

$$= 0.272$$

iii) Let $p_{0.9}$ be the 90^{th} percentile

i.e $P(X \le p_{0.9}) = 0.90$

$$\int_0^{p_{0.9}} \frac{3}{2}(1-x^2)\,dx = 0.9$$

$$\frac{3}{2}\left(p_{0.9} - p_{0.9}^3/3\right) = 0.9 \Leftrightarrow 3p_{0.9} - p_{0.9}^3 - 1.8 =$$

Solve for $p_{0.9}$

2. $Z \sim N(0,1)$

i) $Z_{0.1} = 1.28$

$Z_{0.01} = 2.33$

$Z_{0.001} = 3.09$

$-Z_{0.05} = -1.645$

ii) $P(Z > \underbrace{-2.37}_{z}) = 0.9911$

$P(Z < \underbrace{0.37}_{z}) = 0.6443$

3. Let X denote the burning time

$X \sim N(4.76, (0.04)^2)$

a) i) $P(X < 4.6) = P\left(Z < \frac{4.6 - 4.76}{0.04} = -4\right)$

$= 0.00003$

ii) $P(X > 4.75) = P\left(Z > \frac{4.75 - 4.76}{0.04} = -0.25\right)$

$= 0.5987$

iii) $P(4.62 < X < 4.80) = P(-3.5 < Z < 1)$

$= 0.8413 - 0$ This is an approximation

$= 0.8413$

b) Let $x_{0.75}$ denote the 75^{th} percentile

$P(Z < 0.675) = 0.75$

$$\Rightarrow \frac{x_{0.75} - 4.76}{0.04} = 0.675$$

$x_{0.75} = 4.787$

Interpretation: 75% of all rockets of this kind will burn before 4.787 seconds.

4. $X \equiv$ pull strength ; $X \sim N(10, (1.5)^2)$.
For a sample of size $n = 8$; $\bar{X} \sim N\left(10, \frac{(1.5)^2}{8}\right)$

P(Wire Bonding Process is out of control)

$= P(\bar{X} < 7.75)$

$$= P\left(Z < \frac{7.75 - 10}{\frac{1.5}{\sqrt{8}}} = -4.24\right) = 0.$$

5. $X \equiv$ roll of a disk

$\mu_X = 0.225$ mm ; $\sigma_X = 0.0042$ mm

$\bar{X}$ is the sample mean of a sample of size $n = 40$.

$$C.L.T \Rightarrow \bar{X} \sim N\left(0.225, \frac{(0.0042)^2}{40}\right)$$

$$P(0.2240 \leq \bar{X} \leq 0.2255)$$

$$= P\left(\frac{0.2240 - 0.225}{\frac{0.0042}{\sqrt{40}}} \leq Z \leq \frac{0.2255 - 0.225}{\frac{0.0042}{\sqrt{40}}}\right)$$

$$= P(-1.506 \leq Z \leq 0.753)$$

$$= 0.7744 - 0.0660$$

$$= 0.7084$$

6. $X = \#$ of item that require repair within a year

$X \sim B(1000, 0.02)$; $np = 20$; $np(1-p) = 19.6$

$$P(X \geq 50) = P\left(Z \geq \frac{50 - \frac{1}{2} - 20}{\sqrt{19.6}}\right)$$ Using Normal Approx to Binomial

$$= P(Z \geq 6.66)$$

$$= 0$$

7.

$X \equiv$ decay time in ms

The density of X is $f(x) = \begin{cases} 4e^{-4x} & ; x > 0 \\ 0 & \text{otherwise} \end{cases}$

i) $P(X > 100) = \int_{100}^{\infty} 4e^{-4x}\,dx = e^{-400}$

ii) $P(X < 20) = \int_{0}^{20} 4e^{-4x}\,dx = 1 - e^{-20}$