King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics

Math 102
Exam I
Term 121
Wednesday, Oct. 3, 2012
Net Time Allowed: 120 minutes

MASTER VERSION

1. Using four approximating rectangles and midpoints, the area under the graph of $f(x) = x^2 + 3$ from $x = 0$ to $x = 4$ is approximately equal to

(a) 33

(b) 22

(c) $\frac{81}{4}$

(d) $\frac{17}{4}$

(e) 12

2. Expressing the following limit as a definite integral over the interval $[1, 3]$, we have

$$\lim_{n\to\infty} \sum_{i=1}^{n} x_i^2 e^{x_i} \triangle x =$$

(a) $\int_1^3 x^2 e^x dx$

(b) $\int_1^3 x^2 dx$

(c) $\int_1^3 e^x dx$

(d) $\int_1^3 x e^x dx$

(e) $\int_1^3 (x^2 + e^x) dx$

3. If $F(x) = \int_x^\pi \frac{\cos t}{1+\sqrt{t}}\,dt$, then $F'(\pi) =$

(a) $\dfrac{1}{1+\sqrt{\pi}}$

(b) 0

(c) $\dfrac{-1}{1+\sqrt{\pi}}$

(d) $1+\sqrt{\pi}$

(e) $-\sqrt{\pi}$

4. $\displaystyle\int \frac{6t-1}{2\sqrt{t}}\,dt =$

(a) $2\sqrt{t^3} - \sqrt{t} + C$

(b) $\sqrt{t^3} + 2\sqrt{t} + C$

(c) $3\sqrt[3]{t} - 2\sqrt{t} + C$

(d) $2\sqrt{t^3} - \dfrac{1}{2}\ln\sqrt{t} + C$

(e) $2\sqrt[3]{t^2} - 3\sqrt{t} + C$

5. If the region enclosed by the curves

$$y = \sqrt{x},\ y = \sqrt{2},\ x = 0$$

is rotated about the x- axis, then the volume of the generated solid is equal to

(a) $2\,\pi$

(b) $\pi\,\sqrt{2}$

(c) $\frac{1}{2}\,\pi$

(d) $\frac{1}{3}\sqrt{2}$

(e) $4\,\pi$

6. $\int_{-4}^{4} f(x)dx - \int_{-4}^{-1} f(x)dx + \int_{4}^{7} f(x)dx =$

(a) $\int_{-1}^{7} f(x)dx$

(b) $\int_{-4}^{7} f(x)dx$

(c) $\int_{1}^{4} f(x)dx$

(d) $\int_{-1}^{4} f(x)dx$

(e) $\int_{4}^{7} f(x)dx$

7. Which one of the following statements is **FALSE:** f is a continuous function on the interval $[2, 5]$.

(a) If $f(x) \geq 0$ for $2 \leq x \leq 5$, then $\int_2^5 f(x)dx \geq 3$

(b) If $f(x) \leq 0$ for $2 \leq x \leq 5$, then $\int_2^5 f(x)dx \leq 0$

(c) If $f(x) \leq 1$ for $2 \leq x \leq 5$, then $\int_2^5 f(x)dx \leq 3$

(d) If $2 \leq f(x) \leq 5$ for $2 \leq x \leq 5$, then $6 \leq \int_2^5 f(x)dx \leq 15$

(e) $\int_2^5 f(x)dx + \int_5^2 f(x)dx = 0$

8. Let $y = f(x)$ be the function whose graph is given below:

Then $\int_0^5 f(x)dx =$

(a) -3

(b) 7

(c) 5

(d) -4

(e) 0

9. $\sum_{i=1}^{n} \left(2n - \frac{6\, i^2}{n} \right) =$

(a) $-3n - 1$

(b) $4n^2 + n - 1$

(c) $5n - 2$

(d) $2n - n^2$

(e) $2n^2 + n + 1$

10. A particle moves along a line so that its velocity (m/s) at time t is

$$v(t) = t^3 - 3t^2 + 2t,\ t \geq 0$$

The **total distance** traveled by the particle during the time interval $[1, 3]$ is

(a) $\frac{5}{2}\, m$

(b) $\frac{15}{4}\, m$

(c) $\frac{3}{2}\, m$

(d) $\frac{1}{4}\, m$

(e) $\frac{5}{4}\, m$

11. $\displaystyle\int_{-1}^{1} \frac{1+\sin x}{1+x^2}\, dx =$

(a) $\dfrac{\pi}{2}$

(b) 0

(c) $\pi + \ln 2$

(d) $\ln 2$

(e) $\pi - 1$

12. The area of the region enclosed by the curves $y = x^3 - 2x$ and $y = 2x$ is

(a) 8

(b) 6

(c) 7

(d) 9

(e) 10

13. The area of the region bounded by the curves $x + y = 0$ and $x = y^2 + 3y$ is equal to

(a) $\frac{32}{3}$

(b) $\frac{16}{3}$

(c) $\frac{40}{3}$

(d) $\frac{64}{3}$

(e) $\frac{80}{3}$

14. $\int \frac{dx}{\sqrt{4x - 4x^2}} =$

(a) $\frac{1}{2}\sin^{-1}(2x - 1) + C$

(b) $\sin^{-1}(2x - 1) + C$

(c) $\sin^{-1}(1 - x) + C$

(d) $2\sqrt{4x - 4x^2} + C$

(e) $\frac{x}{2\sqrt{4x - 4x^2}} + C$

15. $\displaystyle\int_0^{\ln(e-1)} \frac{1}{1+e^{-x}} dx =$

(a) $1 - \ln 2$

(b) $2 - \ln 2$

(c) $3 - \ln 2$

(d) $4 - \ln 2$

(e) $5 - \ln 2$

16. $\displaystyle\lim_{n\to\infty} \frac{1}{n}\left[\sqrt{1-\frac{1}{n^2}} + \sqrt{1-\frac{4}{n^2}} + \sqrt{1-\frac{9}{n^2}} + \ldots + \sqrt{1-\frac{n^2}{n^2}}\right] =$

(a) $\dfrac{\pi}{4}$

(b) $\dfrac{\pi}{2}$

(c) $\sqrt{2}$

(d) $1+\sqrt{2}$

(e) π

17. $\int \left(\dfrac{1+\cos^3 x}{1-\sin^2 x} \right) \tan x \, dx =$

(a) $\dfrac{1}{2}\tan^2 x - \cos x + C$

(b) $\dfrac{1}{2}\cos^2 x - \sin x + C$

(c) $\tan x - \ln|1+\cos x| + C$

(d) $\cos x + \ln|1+\tan x| + C$

(e) $\sin x - \cos x + C$

18. If g is a continuous function on $\left[-\dfrac{1}{2}, 2x\right]$ such that

$$\int_{-1/2}^{2x} \cos\left(\frac{t}{2}\right) g(t)dt = -\frac{1}{2} + \frac{1}{2} x \sin x,$$

then

(a) $g(x) = \dfrac{1}{4}\tan\left(\dfrac{x}{2}\right) + \dfrac{x}{8}$

(b) $g(x) = \dfrac{1}{4}\sin\left(\dfrac{x}{2}\right) + \dfrac{x}{8}$

(c) $g(x) = \dfrac{1}{4}\cos\left(\dfrac{x}{2}\right) - \dfrac{x}{8}$

(d) $g(2x) = \dfrac{1}{4}\tan\left(\dfrac{x}{2}\right) + \dfrac{x}{4}$

(e) $g(2x) = \dfrac{1}{4}\tan x + \dfrac{x}{8}$

19. If a is **a positive real number** that satisfies

$$\int_0^{2a} e^x dx = 3\int_0^a e^x dx,$$

then $e^a - 3 =$

(a) -1

(b) 5

(c) 0

(d) 2

(e) -2

20. The base of a solid S is the region enclosed by the curves $y = \dfrac{1}{x}, y = 0, x = 1, x = 3$. If the cross sections of S perpendicular to the x-axis are **semicircles**, then the volume of S is

(a) $\dfrac{\pi}{12}$

(b) $\dfrac{2\pi}{3}$

(c) $\dfrac{\pi}{3}$

(d) $\dfrac{3\pi}{4}$

(e) $\dfrac{\pi}{5}$

15. The area of the region enclosed by the curves $y = x^3 - 2x$ and $y = 2x$ is

(a) 9

(b) 6

(c) 7

(d) 10

(e) 8

16. $\lim_{n\to\infty} \frac{1}{n}\left[\sqrt{1-\frac{1}{n^2}} + \sqrt{1-\frac{4}{n^2}} + \sqrt{1-\frac{9}{n^2}} + \ldots + \sqrt{1-\frac{n^2}{n^2}}\right] =$

(a) $\frac{\pi}{2}$

(b) $1+\sqrt{2}$

(c) $\frac{\pi}{4}$

(d) $\sqrt{2}$

(e) π

17. If g is a continuous function on $\left[-\frac{1}{2}, 2x\right]$ such that

$$\int_{-1/2}^{2x} \cos\left(\frac{t}{2}\right) g(t)dt = -\frac{1}{2} + \frac{1}{2}x \sin x,$$

then

(a) $g(x) = \frac{1}{4}\tan\left(\frac{x}{2}\right) + \frac{x}{8}$

(b) $g(x) = \frac{1}{4}\sin\left(\frac{x}{2}\right) + \frac{x}{8}$

(c) $g(2x) = \frac{1}{4}\tan\left(\frac{x}{2}\right) + \frac{x}{4}$

(d) $g(x) = \frac{1}{4}\cos\left(\frac{x}{2}\right) - \frac{x}{8}$

(e) $g(2x) = \frac{1}{4}\tan x + \frac{x}{8}$

18. $\int \left(\frac{1 + \cos^3 x}{1 - \sin^2 x}\right) \tan x \, dx =$

(a) $\frac{1}{2}\cos^2 x - \sin x + C$

(b) $\frac{1}{2}\tan^2 x - \cos x + C$

(c) $\tan x - \ln|1 + \cos x| + C$

(d) $\cos x + \ln|1 + \tan x| + C$

(e) $\sin x - \cos x + C$

19. The base of a solid S is the region enclosed by the curves $y = \frac{1}{x}, y = 0, x = 1, x = 3$. If the cross sections of S perpendicular to the x-axis are **semicircles**, then the volume of S is

(a) $\frac{\pi}{5}$

(b) $\frac{\pi}{3}$

(c) $\frac{3\pi}{4}$

(d) $\frac{\pi}{12}$

(e) $\frac{2\pi}{3}$

20. If a is **a positive real number** that satisfies

$$\int_0^{2a} e^x dx = 3 \int_0^a e^x dx,$$

then $e^a - 3 =$

(a) -1

(b) 2

(c) -2

(d) 0

(e) 5

Q	MM	V1	V2	V3	V4
1	a	a	e	b	b
2	a	e	a	c	d
3	a	a	a	a	a
4	a	e	e	d	a
5	a	c	a	b	c
6	a	d	c	c	a
7	a	b	e	a	d
8	a	a	c	c	e
9	a	a	d	a	d
10	a	a	b	b	b
11	a	d	c	b	e
12	a	a	e	b	d
13	a	d	b	a	d
14	a	d	e	e	c
15	a	b	b	a	e
16	a	c	b	b	c
17	a	b	e	e	a
18	a	d	b	a	b
19	a	d	a	c	d
20	a	e	d	e	a