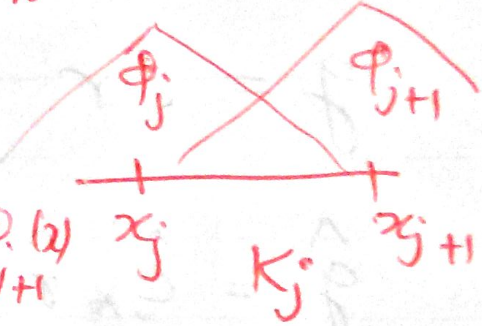
$$\| u - u_h \|_0^2 = \int_0^1 (u - u_h)^2 \, dx$$

$$= \sum_{K_j} \int_{x_j}^{x_{j+1}} (u - u_h)^2 \, dx$$

use mid-point quadrature

$$\cong \sum_{j=1}^{M-1} h \left[u(\bar{x}_j) - u_h(\bar{x}_j) \right]^2 \qquad \bar{x}_j = \frac{x_j + x_{j+1}}{2}$$



$$u_h(x) = u_h(x_j)\,\phi_j(x) + u_h(x_{j+1})\,\phi_{j+1}(x)$$

in the element $K_j = [x_j, x_{j+1}]$

$$u_h(\bar{x}_j) = u_h(x_j)\,\phi_j(\bar{x}_j) + u_h(x_{j+1})\,\phi_{j+1}(\bar{x}_j)$$

but $\phi_j(\bar{x}_j) = \phi_{j+1}(\bar{x}_j) = \frac{1}{2}$

$$u_h(\bar{x}_j) = u_h(x_j)\,\frac{1}{2} + u_h(x_{j+1})\,\frac{1}{2}$$

$$= \frac{1}{2}\left[u_h(x_j) + u_h(x_{j+1}) \right] = \frac{1}{2}\left[v_j + v_{j+1} \right]$$

Hence $\| u - u_h \|_0 \cong \sqrt{\sum_{j=1}^{M-1} h \left[u(\bar{x}_j) - \frac{1}{2}(v_j + v_{j+1}) \right]^2}$