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**MATH-260 Term-092 QUIZ-2a**

1) find the reduced echelon form of the matrix $A = \left[\begin{array}{cccc|c} 2 & 2 & 4 & 2 & 0 \\ 1 & -1 & -4 & 3 & 0 \\ 2 & 7 & 19 & -3 & 0 \end{array}\right]$

$$A \xrightarrow{\frac{1}{2}R_1} \left[\begin{array}{cccc|c} 1 & 1 & 2 & 1 & 0 \\ 1 & -1 & -4 & 3 & 0 \\ 2 & 7 & 19 & -3 & 0 \end{array}\right] \xrightarrow[-2R_1+R_3]{-R_1+R_2} \left[\begin{array}{cccc|c} 1 & 1 & 2 & 1 & 0 \\ 0 & -2 & -6 & 2 & 0 \\ 0 & 5 & 15 & -5 & 0 \end{array}\right]$$

$$\xrightarrow[\frac{1}{5}R_3]{-\frac{1}{2}R_2} \left[\begin{array}{cccc|c} 1 & 1 & 2 & 1 & 0 \\ 0 & 1 & 3 & -1 & 0 \\ 0 & 1 & 3 & -1 & 0 \end{array}\right] \xrightarrow{-R_2+R_3} \left[\begin{array}{cccc|c} 1 & 1 & 2 & 1 & 0 \\ 0 & 1 & 3 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$\xrightarrow{-R_2+R_1} \left[\begin{array}{cccc|c} 1 & 0 & -1 & 2 & 0 \\ 0 & 1 & 3 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

2) Use the method of Gauss-Jordan elimination to solve the following system (find the solution in vector form (i.e) as a linear combination of vectors)

$$A=\left[\begin{array}{cccccc|c} 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & -1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \quad (*)$$

To use Gauss-Jordan elimination to solve the linear system (*), we first transform its augmented matrix into reduced echelon form

$$\left[\begin{array}{cccccc|c} \textcircled{1} & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & \textcircled{1} & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \textcircled{1} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

leading variables

Free variables $x_2, x_4, x_5$

$\textcircled{x_1}\ x_2\ \textcircled{x_3}\ x_4\ x_5\ \textcircled{x_6}$

$x_2 = s,\ x_4 = t,\ x_5 = w$

3ed row $\rightarrow$ $x_6 = 0$

2ed row $\rightarrow$ $x_3 = -t + w$

1st row $\rightarrow$ $x_1 = -t - w$

$$x=\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix}=\begin{bmatrix} -t-w \\ s \\ -t+w \\ t \\ w \\ 0 \end{bmatrix}= t\underbrace{\begin{bmatrix} -1 \\ 0 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}}_{v_1}+s\underbrace{\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}}_{v_2}+w\underbrace{\begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}}_{v_3}$$

The general solution of the linear system in vector form is:

$$x = t\,v_1 + s\,v_2 + w\,v_3$$

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**MATH-260** **Term-092** **QUIZ-2b** and 2d

1) find the reduced echelon form of the matrix $A = \left[\begin{array}{cccc|c} 2 & 2 & 4 & 2 & 0 \\ 3 & 6 & 15 & 0 & 0 \\ 2 & 7 & 19 & -3 & 0 \end{array}\right]$

$\frac{1}{2}R_1 \rightarrow \left[\begin{array}{cccc|c} 1 & 1 & 2 & 1 & 0 \\ 3 & 6 & 15 & 0 & 0 \\ 2 & 7 & 19 & -3 & 0 \end{array}\right] \xrightarrow[-3R_1+R_2]{-2R_1+R_3} \left[\begin{array}{cccc|c} 1 & 1 & 2 & 1 & 0 \\ 0 & 3 & 9 & -3 & 0 \\ 0 & 5 & 15 & -5 & 0 \end{array}\right]$

$\frac{1}{3}R_2 \rightarrow \left[\begin{array}{cccc|c} 1 & 1 & 2 & 1 & 0 \\ 0 & 1 & 3 & -1 & 0 \\ 0 & 1 & 3 & -1 & 0 \end{array}\right] \xrightarrow{-R_2+R_3} \left[\begin{array}{cccc|c} 1 & 1 & 2 & 1 & 0 \\ 0 & 1 & 3 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$

$\frac{1}{5}R_3$

$R_2+R_1 \rightarrow \left[\begin{array}{cccc|c} 1 & 0 & -1 & 2 & 0 \\ 0 & 1 & 3 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$