

King Fahd University of Petroleum and Minerals
Department of Mathematical Sciences
Math 202
First Exam
Semester II, 1999 (982)

Name: _____

ID #: _____

Section (circle one): # 3 (8:00-8:50)

4 (9:00-9:50)

FORM(1)

Notes

- You must show all your work to justify your answer.
- Be as organized as possible.

Problem		points
1		12
2		10
3		10
4		10
5		10
6		12
7		6
8		6
9		6
10		6
11		6
12		6
Total:		100

Problem # 1

(12 points)

State whether the given differential equations are linear or nonlinear. Give the order of each equation.

(a) $x^2 dy + (y - xy - xe^x) dx = 0.$

(b) $\frac{dy}{dx} = \sqrt{1 + \left(\frac{d^2y}{dx^2}\right)^2}.$

(c) $(1 - y^2) dx + x dy = 0.$

Problem # 2

(10 points)

Solve the given differential equation

$$y^2 dx = (3x^4 + 2xy) dy.$$

Problem # 3

(10 points)

Solve the given differential equation

$$\frac{dy}{dx} = -\frac{4}{x^2} - \frac{1}{x}y + y^2,$$

where $y_1 = \frac{2}{x}$ is a particular solution.

Problem # 4

(10 points)

Solve the given differential equation

$$\cos^2(x)\sin(x)dy + (y\cos^3(x) - 1)dx = 0$$

Problem # 5

(10 points)

Solve the given differential equation

$$\frac{dy}{dx} = y + x(y + 1)^2 + 1$$

Problem # 6

(12 points)

Solve the given differential equation

$$y^2 - y - (ye^x + ye^{-x} - \sqrt{y}e^x - \sqrt{y}e^{-x})\frac{dy}{dx} = 0.$$

Problem # 7

(6 points)

If $y(x)$ is the solution of the following initial value problem

$$(x + \sqrt{x})dy = (y + \sqrt{y})dx, \quad y(4) = 0$$

then $y(25) =$ (a) $1/9$ (b) $16/9$ (c) 1 (d) $4/9$ (e) none of the above

...

Problem # 8

(6 points)

If $y(x)$ is the solution of the following initial value problem

$$(4y + 2x - 5)dx + (6y + 4x - 1)dy = 0, \quad y(-1) = 0$$

then $y(x) =$

(a) $4xy + x^2 - 5x + 3y^2 - y = 0$ (b) $4xy + x^2 - 5x + 3y^2 - y = 4$

(c) $4xy + x^2 - 5x + 3y^2 - y = 8$ (d) $4xy + x^2 - 5x + 3y^2 - y = 6$

(e) none of the above

...

Problem # 9

(6 points)

If the following differential equation is exact,

$$(2xy^2 + ye^x)dx + (2x^2y - ke^x - 1)dy = 0, \quad y(-1) = 2$$

then $k =$ (a) 1 (b) 2 (c) 4 (d) 0 (e) none of the above

...

Problem # 10

(6 points)

A thermometer is removed from a room where the air temperature is 70°F to the outside, where the temperature is 10°F . After $\frac{1}{2}$ minute, the thermometer reads 50°F . How long will it take for the thermometer to reach 15°F ?

(a) $t = \frac{\ln 2}{\ln 9 - \ln 4}$ (b) $t = \frac{\ln 3}{\ln 9 - \ln 4}$ (c) $t = \frac{\ln 6}{\ln 9 - \ln 4}$ (d) $t = \frac{\ln 12}{\ln 9 - \ln 4}$ (e) none of the above

...

Problem # 11

(6 points)

A large tank is filled with 500 gallons of pure water. Brine containing 2 lb of salt per gallon is pumped into the tank at rate of 5 gallons per minute. The well-mixed solution is pumped out at the same rate. Find the number of pounds of salt in the tank after 50 min.

(a) $1000(1 - e^{-1})$ (b) $1000(1 - e^{-1/2})$ (c) $1000(1 - e^{-2})$

(d) $1000(1 - e^{-10})$ (e) none of the above

...

Problem # 12

(6 points)

If $y(x)$ is the solution of the following initial value problem

$$y(\ln(x) - \ln(y))dy = (x \ln(x) - x \ln(y) - y)dy, \quad y(1) = e$$

then $y(x) =$ (a) $x = ye^{e/y}$ (b) $x = ye^{1/e y}$ (c) $\ln \left| \ln \frac{x}{y} \right| = \ln |y| - 1$

.

(d) $\ln \left| \ln \frac{x}{y} \right| = -\ln |y| - 2$ (e) none of the above