Name: KEY **ID:** KEY **Sec:** 9AM , 10AM

MATH-102 Term-152 **CQ7** **(show all your work & write final answer in the box)**

1) Write out the form of the partial fraction decomposition of the expression **[DO NOT EVALUATE** the numerical values of the coefficients]

$$\frac{x^{10}+1}{x^3(x^2+3)^2(x^2-3)(x^2+x+1)^3(x-\sqrt{3})} = \frac{x^{10}+1}{x^3(x^2+3)^2(x-\sqrt{3})^2(x+\sqrt{3})(x^2+x+1)^3}$$

$$= \overbrace{\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x^3}}^{2\text{pts}}+\frac{Dx+E}{x^2+3}+\frac{Fx+G}{(x^2+3)^2}+\frac{H}{x-\sqrt{3}}+\frac{I}{(x-\sqrt{3})^2}+\frac{J}{(x+\sqrt{3})}+\frac{Kx+L}{x^2+x+1}$$

2pts each

$$+\frac{Mx+N}{(x^2+x+1)^2}+\frac{Ox+P}{(x^2+x+1)^3}$$

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2) Evaluate the integral $\int \frac{-2x+1}{(x^2+4)(x-1)}\,dx = I$

$$\frac{-2x+1}{(x^2+4)(x-1)} = \frac{Ax+B}{x^2+4}+\frac{C}{x-1}$$ 2

multiply by $(x^2+4)(x-1)$

$$-2x+1 = (Ax+B)(x-1)+C(x^2+4)$$

$\underline{x=1}$: $-1 = 5C \Rightarrow C = -1/5$ 2pts

$\underline{x=0}$: $1 = -B+4C \Rightarrow B = -9/5$ 2

coeff of x^2

$0 = A+C \Rightarrow A = 1/5$ 2

So

$$I = \int \frac{\frac{1}{5}x-\frac{9}{5}}{x^2+4}\,dx - \int \frac{1/5}{x-1}\,dx$$

$$= \frac{1}{5}\int \frac{x}{x^2+4}\,dx - \frac{9}{5}\int\frac{dx}{x^2+4} - \frac{1}{5}\int\frac{dx}{x-1}$$

$$= \frac{1}{5(2)}\int\frac{(2)x\,dx}{x^2+4} - \frac{9}{5}\int\frac{dx}{x^2+4} - \frac{1}{5}\int\frac{dx}{x-1}$$

$$= \frac{1}{10}\ln(x^2+4) - \frac{9}{5}\left(\frac{1}{2}\right)\tan^{-1}\frac{x}{2} - \frac{1}{5}\ln|x-1| + C$$

2 2

$$= \frac{1}{10}\ln(x^2+4) - \frac{9}{10}\tan^{-1}\frac{x}{2} - \frac{1}{5}\ln|x-1| + C$$

2

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3)Evaluate the integral $\int \frac{x^2}{\sqrt{4-x^2}}\,dx = I$

Let $x = 2\sin\theta \quad -\frac{\pi}{2} < \theta \le \frac{\pi}{2}$

$dx = 2\cos\theta\, d\theta$ 2

$\sqrt{4-x^2} = \sqrt{4-4\sin^2\theta} = 2\cos\theta$

$I = \int \frac{4\sin^2\theta}{\cancel{2\cos\theta}}(\cancel{2\cos\theta}\, d\theta)$

$= 4\int \sin^2\theta\, d\theta$ 2

$= 4\int \left(\frac{1}{2} - \frac{1}{2}\cos 2\theta\right)d\theta$

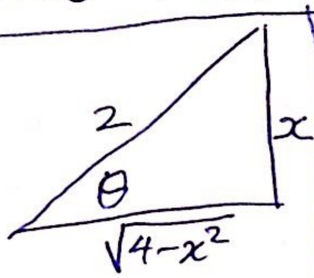
$= 2\int (1-\cos 2\theta)\, d\theta$

$= 2\left[\theta - \frac{1}{2}\sin 2\theta\right] + C$

$= 2\theta - \sin 2\theta + C$ 2

$\sin\theta = \frac{x}{2}$

$\cos\theta = \frac{\sqrt{4-x^2}}{2}$ 2



$I = 2\theta - 2\sin\theta\cos\theta + C$

$= 2\sin^{-1}\left(\frac{x}{2}\right) - 2\left(\frac{x}{2}\right)\left(\frac{\sqrt{4-x^2}}{2}\right) + C$

$= 2\sin^{-1}\left(\frac{x}{2}\right) - \frac{1}{2}x\sqrt{4-x^2} + C$

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Another Solution for (4) 8

$u = 5^x \Rightarrow du = 5^x \ln 5\, dx$

$I = \int \frac{25}{1+5^x}\cdot\frac{5^x\ln 5}{5^x\ln 5}\,dx$ 4

$= \int \frac{25\, du}{(1+u)u\ln 5}\,dx = \frac{25}{\ln 5}\int\frac{du}{u(1+u)}$

$\frac{1}{u(1+u)} = \frac{A}{u} + \frac{B}{1+u} \Rightarrow A=1,\ B=-1$

$I = \frac{25}{\ln 5}\left[\int \frac{1}{u}\,du - \int \frac{du}{1+u}\right]$

$= \frac{25}{\ln 5}\left[\ln|u| - \ln|1+u|\right] + C$ 2

$= \frac{25}{\ln 5}\ln\left|\frac{u}{1+u}\right| + C = \frac{25}{\ln 5}\ln\left|\frac{5^x}{1+5^x}\right| + C$ 4

4)Evaluate the integral $\int \frac{5^2}{1+5^x}\,dx = 25\int\frac{dx}{1+5^x}$

$= 25\int \frac{1}{1+5^x}\cdot\frac{5^{-x}}{5^{-x}}\cdot\frac{(-\ln 5)}{(-\ln 5)}\,dx$

$= 25\int \frac{-5^{-x}\ln 5\, dx}{(5^{-x}+1)(-\ln 5)}$

$= \frac{25}{-\ln 5}\int\frac{-5^{-x}\ln 5}{5^{-x}+1}\,dx$

Let $u = 5^{-x}+1 \Rightarrow$

$du = -5^{-x}\ln 5\, dx$ 4

2

$I = \frac{25}{-\ln 5}\int\frac{du}{u} = \frac{25}{-\ln 5}\ln|u| + C$

$= \frac{25}{-\ln 5}\ln(5^{-x}+1) + C$

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