Name: KEY | **ID:** KEY | **Sec:** 9(9:00-9:50) 10(10:00-10:50)

MATH-102 Term-152 Long Class-QUIZ-10

1) Expand using partial fractions (Do not determine the coefficients): $\dfrac{15x}{(x^5-10x^3)(x^2+10)} = \dfrac{15}{x^2(x^2-10)(x^2+10)}$

$= \dfrac{15}{x^2(x-\sqrt{10})(x+\sqrt{10})(x^2+10)}$

1 1 1 1 2

$= \dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x-\sqrt{10}} + \dfrac{D}{x+\sqrt{10}} + \dfrac{Ex+F}{x^2+10}$

2) Find the average value of the function: $f(x) = \cos^4 x$ on the interval $[0, \pi/4]$

$f_{avg} = \dfrac{1}{\frac{\pi}{4}-0}\int_0^{\pi/4} \cos^4 x\,dx$ 3

$= \dfrac{4}{\pi}\int_0^{\pi/4} \cos^4 x\,dx$

Now, $\cos^4 x = \frac{1}{4}(1+\cos 2x)^2$

$= \frac{1}{4}(1 + 2\cos 2x + \cos^2 2x)$

$= \frac{1}{4}(1 + 2\cos 2x + \frac{1}{2} + \frac{1}{2}\cos 4x)$ 3

$= \frac{3}{8} + \frac{1}{2}\cos 2x + \frac{1}{8}\cos 4x$

$\int \cos^4 x\,dx = \int (\frac{3}{8} + \frac{1}{2}\cos 2x + \frac{1}{8}\cos 4x)\,dx$

$= \frac{3}{8}x + \frac{1}{4}\sin 2x + \frac{1}{32}\sin 4x + C$

$\int_0^{\pi/4} \cos^4 x\,dx = \frac{3}{8}\left(\frac{\pi}{4}\right) + \frac{1}{4}(1) + \frac{1}{32}(0) - 0$

$= \frac{3\pi}{32} + \frac{1}{4}$ 3

Hence, $f_{avg} = \frac{4}{\pi}\left(\frac{3\pi}{32} + \frac{1}{4}\right)$

$= \frac{3}{8} + \frac{1}{\pi}$ 2

3) Evaluate the improper integral $\displaystyle\int_0^4 \frac{dx}{\sqrt{4-x}} = I$ 1

$f(x) = \dfrac{1}{\sqrt{4-x}}$ has an infinit discont at $x=4$ 2

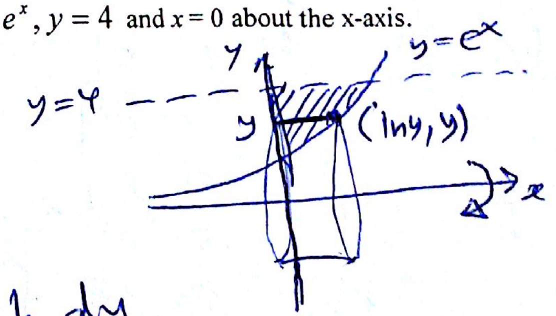
$I = \lim_{t\to 4^-} \int_0^t \dfrac{dx}{\sqrt{4-x}} = \lim_{t\to 4^-} \left[(4-x)^{1/2}(-2)\right]_0^t$ 2

$= -2\lim_{t\to 4^-} \left[\sqrt{4-x}\right]_0^t = -2\lim_{t\to 4^-}\left[\sqrt{4-t} - 2\right]$

$= -2[0-2] = 4$ 4

Hence, the improper integral is conv and its value is 4 2

4) Using shells method, set up the integral (DONOT EVALUATE) The region bounded by the curves $y = e^x$, $y = 4$ and $x = 0$ about the x-axis.



radius $= y$

$h = \ln y$

$V = \int_1^4 2\pi r h\,dy$

$= \int_1^4 2\pi y \ln y\,dy$

3 3 1 3

5) Find the exact area of the surface obtained by rotating the curve $x = \frac{1}{3}(y^2+2)^{3/2}$, $1 \le y \le 2$ about the x–axis

$$\frac{dx}{dy} = \frac{1}{2}(y^2+2)^{1/2}(2y)$$

$$\sqrt{1+\left(\frac{dx}{dx}\right)^2} = \sqrt{1+y^2(y^2+2)} = \sqrt{y^4+2y^2+1}$$

$$= \sqrt{(y^2+1)^2} = y^2+1$$

$$S = \int_1^2 2\pi y \sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy \quad 3$$

$$= \int_1^2 2\pi(y^3+y)\, dy \quad 3$$

$$= 2\pi\left[\frac{1}{4}y^4 + \frac{1}{2}y^2\right]_1^2$$

$$= 2\pi\left[(4+2) - \left(\frac{1}{4}+\frac{1}{2}\right)\right]$$

$$= 2\pi\left[6 - \frac{3}{4}\right] = 2\pi\left[\frac{21}{4}\right]$$

$$= \frac{21\pi}{2} \quad 4$$

6) Evaluate: $\int \frac{x^2\,dx}{\sqrt{9-x^2}} = I$ 3

$$x = 3\sin\theta \Rightarrow dx = 3\cos\theta\, d\theta$$

$$\sqrt{9-x^2} = 3\sqrt{1-\sin^2\theta} = 3\cos\theta$$

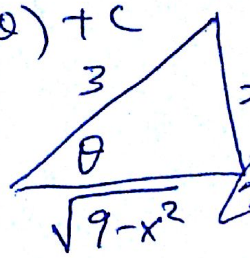
$$I = \int \frac{9\sin^2\theta}{3\cos\theta}(3\cos\theta)\, d\theta \quad 2$$

$$= \int 9\sin^2\theta\, d\theta = 9\int \sin^2\theta\, d\theta$$

$$= 9\int\left(\frac{1}{2} - \frac{1}{2}\cos 2\theta\right) d\theta$$

$$= 9\left[\frac{1}{2}\theta - \frac{1}{4}\sin 2\theta\right] + c$$

$$= \frac{9}{2}\theta - \frac{9}{4}(2\sin\theta\cos\theta) + c \quad 3$$



2

$$= \frac{9}{2}\theta + \frac{9}{2}\sin\theta\cos\theta + c$$

$$= \frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right) + \frac{9}{2}\left(\frac{x}{3}\right)\left(\frac{\sqrt{9-x^2}}{3}\right) + c$$

$$= \frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right) + \frac{1}{2}x\sqrt{9-x^2} + c$$

3

7) Evaluate: $\int (1+x)^2 e^x \tan^{-1} x\,dx = I$

$$u = \tan^{-1} x \qquad dv = (1+x)^2 e^x\, dx$$

$$du = \frac{dx}{1+x^2} \quad 5 \qquad v = e^x\left[(1+x)^2 - 2(1+x) + 2\right]$$

$$= e^x\left[x^2+1\right]$$

$(1+x)^2$	$+$	e^x
$2(1+x)$	$-$	e^x
2	$+$	e^x
0		e^x

$$I = uv - \int v\, du$$

$$= e^x \tan^{-1}x\,(x^2+1) - \int e^x\, dx$$

$$= e^x \tan^{-1}x\,(x^2+1) - e^x + c$$

$$\boxed{\int (1+x)^2 e^x \tan^{-1} x\, dx = (x^2+1)e^x \tan^{-1} x - e^x + c}$$

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