Name: KEY	ID: KEY	Sec: 10

MATH-102 Term-142 Class-QUIZ-on-Series

(show all your work and circle one letter to get a full mark or you will get zero)

1)The series $\sum_{n=1}^{\infty} \frac{(-n!)^4}{(4n+12)!}$ is

$= \sum \frac{(-1)^4 \cdot (n!)^4}{(4n+12)!} = \sum \frac{(n!)^4}{(4n+12)!}$

a)conditionally convergent
b)a divergent p series
c)divergent by the ratio test
d)a series for which the ratio test is inconclusive
e) absolutely convergent
f)none of the above

$$\frac{a_{n+1}}{a_n} = \frac{[(n+1)!]^4}{(4n+16)!} \cdot \frac{(4n+12)!}{[n!]^4}$$

$$= \frac{(n+1)^4}{(4n+16)(4n+15)(4n+14)(4n+13)} \to \frac{1}{4^4} < 1$$

2)The series $\sum_{n=1}^{\infty} \frac{3^{n-1} n^n}{2^{2n+3}}$ is

$= \sum \frac{3^{-1} \cdot 3^n \cdot n^n}{2^{2n} \cdot 2^3} = \frac{1}{24} \sum_{n=1}^{\infty} \left(\frac{3n}{4}\right)^n$

a)diverges by the root test
b)a convergent p series
c)converges by the root test
d)a series for which the root test is inconclusive
e) a divergent geometric series
f)none of the above

by root test

$$\lim (a_n)^{1/n} = \lim_{n\to\infty} \frac{3n}{4} = \infty > 1$$

divg

3)The series $\sum_{n=1}^{\infty} (-1)^n (\sqrt{n+3} - \sqrt{n+2})$ is

a)diverges by the limit comparison test
b) conditionally convergent
c) absolutely convergent
d)diverges by the divergent test
e) divergent by the ratio test
f)none of the above

$\sum |a_n| = \sum (\sqrt{n+3} - \sqrt{n+2})$

multiply by conjugate

$$\sum (\sqrt{n+3} - \sqrt{n+2}) = \sum \frac{1}{\sqrt{n+3} + \sqrt{n+2}}$$

Compare $\sum \frac{1}{\sqrt{n}}$

$$\lim_{n\to\infty} \frac{\frac{1}{\sqrt{n+3}+\sqrt{n+2}}}{\frac{1}{\sqrt{n}}} = \lim \frac{\sqrt{n}}{\sqrt{n+3}+\sqrt{n+2}} = \frac{1}{2}$$

both divg

by alt. test series: $\lim = 0$, alt, $f' < 0$

4)The series $\sum_{n=1}^{\infty} \frac{1}{(\sqrt{n+1} - 1)\sqrt{n+1}}$ is

$= \sum \frac{1}{(n+1) - \sqrt{n+1}}$

a)diverges by the limit comparison test
b) convergent by the integral test
c) convergent by the ratio test
d) convergent by the root test
e) divergent by the ratio test
f)none of the above

Compare with $\sum \frac{1}{n}$

$$\lim \frac{\frac{1}{(n+1)-\sqrt{n+1}}}{1/n} = \lim \frac{n}{(n+1)-\sqrt{n+1}} = \lim \frac{1}{1+\frac{1}{n} - \sqrt{\frac{1}{n}+\frac{1}{n^2}}} = 1$$

both divg

5)The series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{4^{n+2} + (n+3)^3}$ is

$\sum |a_n| = \sum \frac{1}{4^{n+2} + (n+3)^3}$

a)conditionally convergent
b)divergent
c)absolutely convergent
d)convergent by the integral test
e)divergent by the alternating series test
f)none of the above

$$4^{n+2} + (n+3)^3 > (n+3)^3 > n^3$$

$$\Rightarrow \frac{1}{4^{n+2} + (n+3)^3} < \frac{1}{n^3}$$

Conv

6)The series $\sum_{n=1}^{\infty} (3\sqrt{3} - \sqrt[n]{3})^{\frac{n}{2}}$ is

a)the root test is inconclusive
b) conditionally convergent
c) a divergent geometric series
d) convergent by the root test
e) divergent by the root test
f)none of the above

$$(a_n)^{1/n} = \left(3\sqrt{3} - 3^{1/n}\right)^{1/2}$$

$$\lim (a_n)^{1/n} = (3\sqrt{3} - 0)^{1/2} = \sqrt{3\sqrt{3}} > 1$$

divg by root test

7)If the sum of the first n terms of a series $\sum_{n=1}^{\infty} a_n$ is given by

$S_n = \dfrac{2n}{n+2}$ then $a_9 =$

a) 1/110
b)4/110
c)2/120
d)2/101
e) 2/101
f)none of the above

$$a_n = S_n - S_{n-1}$$

$$a_9 = S_9 - S_8$$

$$= \frac{18}{11} - \frac{16}{10}$$

$$= \frac{180-176}{110} = \frac{4}{110}$$

8) By applying the ratio test to the series $\sum_{n=0}^{\infty} \dfrac{\sqrt{1+n}}{1+(1+n)^2}$ we conclude that

a)conditionally convergent
b)divergent
c)absolutely convergent
d)convergent
e)the test is inconclusive
f)none of the above

$$\frac{a_{n+1}}{a_n} = \frac{\sqrt{2+n}}{1+(2+n)^2} \cdot \frac{1+(1+n)^2}{\sqrt{1+n}}$$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim \sqrt{\frac{2+n}{1+n}} \cdot \frac{1+(1+n)^2}{1+(2+n)^2}$$

$$= \lim \sqrt{\frac{2+n}{1+n}} \cdot \lim \frac{1+(1+n)^2}{1+(2+n)^2}$$

$$= (1)\cdot(1) = 1$$

9)The series $\sum_{n=1}^{\infty} \dfrac{(-1)^{n+1} n}{n^3+1}$ is

a)conditionally convergent
b)divergent
c)absolutely convergent
d)convergent by the integral test
e)divergent by the alternating series test
f)none of the above

Study $\sum |a_n| = \sum \frac{n}{n^3+1}$

Compare with $\sum \frac{1}{n^2}$

$$\lim \left(\frac{n}{n^3+1} \div \frac{1}{n^2} \right)$$

$$= \lim \frac{n^3}{n^3+1} = 1$$

both conv

Hence, AC

10)The series $\sum_{n=1}^{\infty} \dfrac{(-1)^n (n!)^2 3^n}{(n+1)!}$ is

a)conditionally convergent
b)divergent
c)absolutely convergent
d)convergent by the integral test
e)divergent by the alternating series test
f)none of the above

Study $\sum \frac{(n!)^2 3^n}{(n+1)!}$

ratio test

$$\frac{a_{n+1}}{a_n} = \frac{((n+1)!)^2 3^{n+1}}{(n!)^2 \cdot 3^n} \cdot \frac{(n+1)!}{(n+2)!}$$

$$= \frac{(n+1)^2 \cdot 3}{1} \cdot \frac{1}{(n+2)} \to \infty$$

$\sum |a_n|$ is divg

$$\lim a_n = \lim \frac{(n!)^2 3^n}{n+1} = \infty$$

Hence divg

11)The series $\sum_{n=1}^{\infty} \dfrac{n^2 + n\ln n}{1+2\ln n}$ is

a)conditionally convergent
b)divergent
c)absolutely convergent
d)convergent by the integral test
e)divergent by the alternating series test
f)none of the above

$$\lim a_n \overset{LH}{=} \lim \frac{2n + \ln n + 1}{2/n} \quad \text{(multiply by } n\text{)}$$

$$= \lim \frac{2n^2 + n\ln n + n}{2} = +\infty \neq 0$$

Hence divg by nth term divg test

12)The series $\sum_{n=1}^{\infty} \dfrac{(-1)^{n-1} \cos n}{(n+3)^4}$ is

a)conditionally convergent
b)divergent
c)absolutely convergent
d)divvergent by the root test
e)divergent by the alternating series test
f)none of the above

study $\sum |a_n|$

$$\cos n \le 1$$

$$\frac{\cos n}{(n+3)^4} \le \frac{1}{(n+3)^4}$$

Also, $\dfrac{\cos n}{(n+3)^4} \le \dfrac{1}{(n+3)^4} \le \dfrac{1}{n^4}$

p-series conv