120 10 each

Name: KEY **ID:** KEY **Sec:** 8 (9:00-9:50) 9 (10:00-10:50)

MATH-102 Term-152 Long-QUIZ-on-Series

(show all your work and circle one letter to get a full mark or you will get zero)

1) $\sum_{n=1}^{\infty}\frac{(-1)^n 2^{n-1}}{4^{n+1}} = \sum_{n=1}^{\infty}\frac{(-1)^n 2^n \cdot 2^{-1}}{4^n \cdot 4}$

(a) -13/45
(b) -1/256
(c) -1/48
(d) 2/45
(e) -1/2
(f) none of the above

$= \sum_{n=1}^{\infty}\left(\frac{-2}{4}\right)^n \frac{1}{8} = \frac{1}{8}\sum_{n=1}^{\infty}\left(-\frac{1}{2}\right)^n$

geometric with

$r = -1/2 \quad a = -1/16$

$Sum = \frac{a}{1-r} = \frac{-1/16}{1+\frac{1}{2}} = \frac{-1/16}{3/2}$

$= \frac{-1}{24}$

4) If the sum of the first n terms of a series $\sum_{n=1}^{\infty} a_n$ is given by $S_n = \frac{2n}{n+1}$ then $a_{11} = S_{11} - S_{10}$

a) 22/12
b) 4/110
c) 1/33
d) 1/101
e) 1/66
f) none of the above

$= \frac{22}{12} - \frac{20}{11}$

$= \frac{11}{6} - \frac{20}{11} = \frac{121-120}{66}$

$a_n = S_n - S_{n-1} \qquad = 1/66$

2) if $\{S_n\}$ is the sequence of partial sums of the series $\sum_{n=1}^{\infty}\frac{1}{(n+1)(n+2)}$ then $S_{10} =$

(a) 3/10
(b) 5/12
(c) 1/(11x12)
(d) 10
(e) 1/12
(f) none of the above

$\frac{1}{(n+1)(n+2)} = \frac{1}{n+1} - \frac{1}{n+2}$

$\sum_{n=1}^{\infty}\left(\frac{1}{n+1} - \frac{1}{n+2}\right)$ telescoping

$S_{10} = \underbrace{\left(\frac{1}{2} - \cancel{\frac{1}{3}}\right)}_{n=1} + \underbrace{\left(\cancel{\frac{1}{3}} - \cancel{\frac{1}{4}}\right)}_{n=2} + \cdots + \underbrace{\left(\cancel{\frac{1}{10}} - \cancel{\frac{1}{11}}\right)}_{n=9} + \underbrace{\left(\cancel{\frac{1}{11}} - \frac{1}{12}\right)}_{n=10}$

$= \frac{1}{2} - \frac{1}{12} = \frac{6-1}{12} = 5/12$

5) If n is the smallest number of terms that are required to ensure that the sum of the series $\sum_{n=1}^{\infty}\frac{2}{n^3}$ is accurate within 0.0001 then n equals to

a) 99
b) 100
c) 101
d) 9
e) 10
f) none of the above

the series is with positive term

$\int_n^{\infty}\frac{2}{x^3}\,dx \le \frac{1}{10000}$

$\lim_{t\to\infty}\left[-\frac{1}{x^2}\right]_n^t \le \frac{1}{10000}$

$\frac{1}{n^2} \le \frac{1}{10000}$

$n^2 \ge 10000$

$n \ge 100$

3) The series $\sum_{n=5}^{\infty}\left(\frac{1}{n+2} - \frac{1}{n}\right)$ is

(a) -11/30
(b) 33/42
(c) 1/3
(d) -1/33
(e) 1
(f) none of the above

$= -\sum_{n=5}^{\infty}\left(\frac{1}{n} - \frac{1}{n+2}\right)$

$= -\sum_{n=5}^{\infty}\left(\frac{1}{n} - \frac{1}{n+1} + \frac{1}{n+1} - \frac{1}{n+2}\right)$

$= -\left[\sum_{n=5}^{\infty}\left(\frac{1}{n} - \frac{1}{n+1}\right) + \sum_{n=5}^{\infty}\left(\frac{1}{n+1} - \frac{1}{n+2}\right)\right]$

two telescoping seriess

$= -\left[\frac{1}{5} + \frac{1}{6}\right] = -\left[\frac{6+5}{30}\right]$

$= -11/30$

6) If n is the smallest number of terms that are required to approximate the sum of the series $\sum_{n=2}^{\infty}\frac{(-1)^{n-1}}{n^3-1}$ so that the error is less than 0.001 then n equals to

a) 8
b) 100
c) 20
d) 12
e) 10
f) none of the above

alt. series

$u_{n+1} < 0.001$

$\frac{1}{(n+1)^3 - 1} < \frac{1}{1000}$

$(n+1)^3 - 1 > 1000$

$(n+1)^3 > 1001$

$n+1 > 10 + \text{small number}$

$n > 9 + \text{small number}$

$\Rightarrow n = 10$

7)The series $\sum_{n=1}^{\infty}\frac{(-n!)^4}{(4n+3)!}$ is $= \sum_{n=1}^{\infty}\frac{(n!)^4}{(4n+3)!}$

ratio test

a)conditionally convergent
b)a divergent p series
c)divergent by the ratio test
d)a series for which the ratio test is inconclusive
e) absolutely convergent
f)none of the above

$\frac{[(n+1)!]^4}{(4n+7)!}\cdot\frac{(4n+3)!}{[n!]^4}$

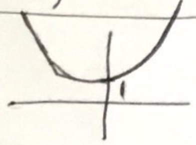
$\lim_{n\to\infty} = \frac{1}{4^4} < 1 \Leftarrow = \frac{(n+1)^4}{(4n+7)(4n+6)(4n+5)(4n+3)}$

abs conv

10) The series $\sum_{n=1}^{\infty}\frac{(4-\cosh^2 n)}{n\sqrt[4]{n}}$ is

W

y = cosh x



(a) divergent by ratio test.
(b) diverges by the divergence test.
(c) convergent by comparison test.
(d) divergent by the integral test.
(e) convergent by the ratio test.
(f) none of the above

$\cosh x \geq 1$

$\cosh^2 x \geq 1$

$-\cosh^2 x \leq -1$

if $n > 1$ $4-\cosh^2 n$ is negative

$W = -\sum_{n=1}^{\infty}\frac{\cosh^2 n - 4}{n\sqrt[4]{n}}$ → series with positive terms

$\lim_{n\to\infty}\frac{\cosh^2 n - 4}{n^{5/4}} \overset{LH}{=} \lim_{n\to\infty}\frac{2(\cosh n)(\sinh n)}{\frac{5}{4}n^{1/4}} = +\infty$ divg

8)The series $\sum_{n=1}^{\infty}\frac{3^n n^n}{2^{2n+1}}$ is

$a_n = \frac{3^n n^n}{2^{2n}\cdot 2} = \frac{3^n\cdot n^n}{4^n\cdot 2} = \left(\frac{3n}{4}\right)^n\frac{1}{2}$

a)diverges by the root test
b)a convergent p series
c)converges by the root test
d)a series for which the root test is inconclusive
e) a divergent geometric series
f)none of the above

root test

$\lim_{n\to\infty}\left[\left(\frac{3n}{4}\right)^n\frac{1}{2}\right]^{1/n}$

$= \lim_{n\to\infty}\frac{3n}{4}\cdot\frac{1}{2^{1/n}} = +\infty > 1$ divg

11)The series $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{3^{n+1}+(n+3)^3}$ is

$3^{n+1} + (n+3)^3 > (n+3)^3 > n^3$

a)conditionally convergent
b)divergent
c)absolutely convergent
d)convergent by the integral test
e)divergent by the alternating series test
f)none of the above

$\Rightarrow \frac{1}{3^{n+1}+(n+3)^3} < \frac{1}{n^3}$ ← convg

by comparison test

$\sum_{n=1}^{\infty}\frac{1}{3^{n+1}+(n+3)^3}$ convg

$\Rightarrow$ our series is AC

9) The series $\sum_{n=2}^{\infty}(-1)^n\frac{1}{n(\ln n)^2}$ is

(a) absolutely convergent..
(b) conditionally convergent.
(c) convergent as its sum is zero.
(d) divergent by the alternating series test.
(e) convergent as its sum is *ln 2*
(f) none of the above

$\int\frac{dx}{x(\ln x)^2}$ $u = \ln x$ $du = \frac{1}{x}dx$

$= \int\frac{du}{u^2} = -\frac{1}{\ln x} + C$

$\int_2^{\infty}\frac{dx}{x(\ln x)^2} = \left[-\frac{1}{\ln x}\right]_2^{\infty}$

$= \lim_{t\to\infty}\left[-\frac{1}{\ln t} + \frac{1}{\ln 2}\right] = \frac{1}{\ln 2}$

Hence, $\sum_{n=2}(-1)^n\frac{1}{n(\ln n)^2}$ is AC

12)The series $\sum_{n=1}^{\infty}(5\sqrt{5}-\sqrt[n]{5})^{\frac{n}{2}}$ is

a)the root test is inconclusive
b) conditionally convergent
c) a divergent geometric series
d) convergent by the root test
e) divergent by the root test
f)none of the above

root test

$\lim_{n\to\infty}|a_n|^{1/n} = \lim_{n\to\infty}\left(5\sqrt{5}-5^{1/n}\right)^{1/2}$

$= \left(5\sqrt{5}-5^0\right)^{1/2} = \left(5\sqrt{5}-1\right)^{1/2}$

$= \sqrt{5\sqrt{5}-1} > 1$