

(show all your work and circle one letter to get a full mark or you will get zero)

1) The Taylor series for the function $f(x) = \sqrt{x}$ about $a = 1$ is given by $f(x) = x^{1/2}$

(a) $1 - \frac{1}{2}(x-1) + \frac{1}{4}(x-1)^2 + \frac{3}{8}(x-1)^3 + \dots$ $f'(x) = \frac{1}{2}x^{-1/2}$

(b) $1 + \frac{1}{2}(x-1) - \frac{1}{8}(x-1)^2 + \frac{1}{16}(x-1)^3 + \dots$ $f''(x) = -\frac{1}{4}x^{-3/2}$

(c) $1 + \frac{1}{2}(x-1) - \frac{1}{4}(x-1)^2 + \frac{5}{8}(x-1)^3 + \dots$

(d) $1 + \frac{1}{2}(x-1) + (x-1)^2 + \frac{3}{8}(x-1)^3 + \dots$ $f(1) = 1$

(e) $1 + (x-1) - \frac{1}{4}(x-1)^2 + \frac{3}{8}(x-1)^3 + \dots$ $f'(1) = \frac{1}{2}$

(f) none of the above $C_2 = \frac{-1/4}{2!} = -\frac{1}{8}$ $f''(1) = -\frac{1}{4}$

2) Using the binomial series, we have, for $|x| < \frac{1}{2}$

$\sqrt{4+32x^3} = (4+32x^3)^{1/2} = (4)^{1/2}(1+8x^3)^{1/2}$
 $= 2(1+8x^3)^{1/2}$ $R = 1/2$

(a) $1 + \frac{1}{4}x^3 + 8x^6 + 32x^9 + \dots$

(b) $1 + \frac{1}{4}x^3 - 8x^6 + 32x^9 + \dots$

(c) $2 + 8x^3 + 16x^6 + 64x^9 + \dots = 1 + \frac{1}{2}(8x^3) + \frac{1}{2}(\frac{-1}{2})(8x^3)^2 + \dots$

(d) $2 - 8x^3 - 16x^6 - 64x^9 + \dots$

(e) $2 + 8x^3 - 16x^6 + 64x^9 + \dots = 1 + 4x^3 - 8x^6 + \dots$

(f) none of the above $\sqrt{4+32x^3} = 2 + 8x^3 - 16x^6 + \dots$

3) The power series representation for the function $f(x) = \frac{9x^2}{2+6x^2}$ is $= \frac{9x^2}{2} \frac{1}{1+3x^2}$

(a) $\sum_{n=0}^{\infty} (-1)^n 3^n x^{2n}$ $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$ $x \rightarrow -3x^2$

(b) $\sum_{n=0}^{\infty} (-1)^n 3^{n+2} x^{2n+2}$ $\frac{1}{1+3x^2} = \sum_{n=0}^{\infty} (-1)^n 3^n x^{2n}$

(c) $\sum_{n=0}^{\infty} \frac{(-1)^n 3^{n+2} x^{2n+2}}{2}$ multiply by $\frac{9x^2}{2}$

(d) $\sum_{n=0}^{\infty} (-1)^n 3^{n+1} x^{2n+2}$ $\frac{9x^2}{2(1-x)} = \sum_{n=0}^{\infty} \frac{(-1)^n 3^{n+2} x^{2n+2}}{2}$

(e) $\frac{9}{2} \sum_{n=1}^{\infty} (-1)^n 3^n x^{2n+2}$

(f) none of the above

4) if the Maclaurin series of $e^x \sin x$ is $A + Bx + Cx^2 + Dx^3 + \dots$

then $C+D =$

(a) 1/2

(b) 4/3

(c) 5/6

(d) 0

(e) 1

(f) none of the above

$e^x \sin x = (1+x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \dots) * (\sin x)$

Coeff of $x^2 \Rightarrow x^2 \Rightarrow C = 1$

Coeff of $x^3 \Rightarrow (1)(-\frac{1}{6})x^3 + \frac{1}{2}x^2(x) \Rightarrow D = -\frac{1}{6} + \frac{1}{2} = \frac{1}{3}$

$\Rightarrow C+D = 1 + \frac{1}{3} = \frac{4}{3}$

5) $\int \frac{1}{x^2} \cos(x^3) dx =$

(a) $c + \sum_{n=0}^{\infty} \frac{(-1)^n x^{6n-1}}{(2n)!(6n-1)}$

(b) $c + \sum_{n=0}^{\infty} \frac{(-1)^n x^{6n-2}}{(2n)!(6n-2)}$

(c) $c + \sum_{n=0}^{\infty} \frac{(-1)^n x^{6n-1}}{(2n+1)!(6n-1)}$

(d) $c + \sum_{n=3}^{\infty} \frac{(-1)^n x^{6n-1}}{(2n)!(6n-1)}$

(e) $c + \sum_{n=0}^{\infty} \frac{(-1)^n x^{6n-1}}{(2n)!(3n-1)}$

(f) none of the above

$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$

$\cos x^3 = \sum_{n=0}^{\infty} \frac{(-1)^n x^{6n}}{(2n)!}$

$\frac{1}{x^2} \cos x^3 = \sum_{n=0}^{\infty} \frac{(-1)^n x^{6n-2}}{(2n)!}$

$\int \frac{1}{x^2} \cos x^3 dx = \sum_{n=0}^{\infty} \frac{(-1)^n x^{6n-1}}{(2n)! (6n-1)}$

6) Let $g(x) = x^3 \tan^{-1} x$ and let $g''(x) = \sum_{n=0}^{\infty} c_n x^n$

then $c_{10} =$

(a) 149/81

(b) 153/17

(c) 132/9

(d) 232/17

(e) 137/89

(f) none of the above

$\tan^{-1} x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}$

$g = x^3 \tan^{-1} x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+4}}{2n+1}$

$g''(x) = \sum_{n=0}^{\infty} \frac{(-1)^n (2n+4)(2n+3) x^{2n+2}}{(2n+1)}$

$2n+2 = 10 \Rightarrow n = 4$

when $n = 4 \Rightarrow \frac{(-1)^4 (12)(11)}{(9)} x^{10} = \frac{132}{9} x^{10}$

$= c + \sum_{n=0}^{\infty} \frac{(-1)^n x^{6n-1}}{(2n)! (6n-1)}$

$= \frac{132}{9} x^{10}$

7) $\sum_{n=1}^{\infty} \frac{n}{2^{n-1}} = \frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$ diff

(a) 9
 (b) 1/7
 (c) 13/9
 (d) 22
 (e) 4
 (f) none of the above

$\frac{1}{(1-x)^2} = \sum_{n=1}^{\infty} n x^{n-1}$ let $x = \frac{1}{2}$

$\frac{1}{(1-\frac{1}{2})^2} = \sum_{n=1}^{\infty} \frac{n}{2^{n-1}}$

4 =

8) Let $g(x) = (1+x^3)^{2/3}$ and let $g''(x) = \sum_{n=0}^{\infty} c_n x^n$

then $c_{10} = k = 2/3$
 the term containing x^{12} in $g(x)$ is

(a) -308/81
 (b) -77/27
 (c) -44/9
 (d) 0
 (e) 2
 (f) none of the above

$(\frac{2}{3})(\frac{2}{3}-1)(\frac{2}{3}-2)(\frac{2}{3}-3)(x^3)^4$

coeff of x^{10} in $g''(x)$ is

$(\frac{2}{3})(-\frac{1}{3})(-\frac{4}{3})(-\frac{7}{3}) \times \frac{4}{2} \times 11$

$\frac{4 \times 3 \times 2 \times 1}{81} = -\frac{11 \times 7 \times 4}{81}$

9) The interval of convergence of the power series $\sum_{n=1}^{\infty} (\sqrt{n+1} - \sqrt{n})(x-5)^n$ is

(a) [2,8]
 (b) (4,6)
 (c) (4,6]
 (d) [4,6]
 (e) (2,8]
 (f) none of the above

$\left| \frac{a_{n+1}}{a_n} \right| = \left| \frac{(\sqrt{n+2} - \sqrt{n+1})(x-5)^{n+1}}{(\sqrt{n+1} - \sqrt{n})(x-5)^n} \right|$

$= \left| \frac{\sqrt{n+2} - \sqrt{n+1}}{\sqrt{n+1} - \sqrt{n}} (x-5) \right|$

$\Rightarrow R = 1$

endpoints: $x=4 \rightarrow$ conv
 $x=6 \rightarrow$ diverg

10) $\frac{1}{1 \cdot 2} - \frac{1}{3 \cdot 2^3} + \frac{1}{5 \cdot 2^5} - \frac{1}{7 \cdot 2^7} + \dots =$

(a) $\tan^{-1}(\frac{1}{2})$
 (b) $\ln(\frac{5}{2})$
 (c) $e^{-1/2}$
 (d) $\sin(\frac{3}{2})$
 (e) $-\frac{3}{2} \cos(\frac{3}{2})$
 (f) none of the above

$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots$

let $x = \frac{1}{2}$

$\tan^{-1} \frac{1}{2} = \frac{1}{2} - \frac{1}{3 \cdot 2^3} + \frac{1}{5 \cdot 2^5} - \frac{1}{7 \cdot 2^7} + \dots$

11) The radius of convergence of the power series $\sum_{n=0}^{\infty} [1 \cdot 3 \cdot 5 \cdot 7 \dots (2n+1)] (2x-5)^{2n}$ equals

(a) ∞
 (b) 5/2
 (c) 1
 (d) 0
 (e) π
 (f) none of the above

$\left| \frac{a_{n+1}}{a_n} \right| = \left| \frac{1 \cdot 3 \cdot 5 \cdot 7 \dots (2n+3)(2x-5)^{2n+2}}{1 \cdot 3 \cdot 5 \dots (2n+1)(2x-5)^{2n}} \right|$

$= (2n+3) |2x-5|^2 \rightarrow \infty$ as $n \rightarrow \infty$

$\Rightarrow R = 0$

12) The sum of the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n 3^{n+1}}$ is equal

to

(a) $\ln(4/3)$
 (b) $-\ln(4/3)$
 (c) $1/3 - \ln(4/3)$
 (d) $1/9 - \ln^3 \sqrt{4/3}$
 (e) $1/9 - \ln^2 \sqrt{4/3}$
 (f) none of the above

$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^n}{n}$

let $x = 1/3$

$\ln(4/3) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} (1/3)^n}{n 3^n}$

$-\ln(4/3) = \sum_{n=1}^{\infty} \frac{(-1)^n}{n 3^n}$

multiply by $\frac{1}{3}$

13) $(e-2) - \frac{(e-2)^2}{2} + \frac{(e-2)^3}{3} - \frac{(e-2)^4}{4} + \frac{(e-2)^5}{5} - \dots =$

(a) $\ln(e-1)$
 (b) $\ln(1-e)$
 (c) $\ln(e-2)$
 (d) $\ln(2-e)$
 (e) $2 \ln(e-1)$
 (f) none of the above

$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$

let $x = e-2$

$\ln(e-1) = (e-2) - \frac{(e-2)^2}{2} + \frac{(e-2)^3}{3} - \frac{(e-2)^4}{4} + \dots$

$-\frac{1}{3} \ln(4/3) = \sum_{n=1}^{\infty} \frac{(-1)^n}{n 3^{n+1}}$

$\ln^3 \sqrt{3/4} =$

#9) $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = |x-5| \cdot \lim_{n \rightarrow \infty} \frac{\sqrt{n+2} - \sqrt{n+1}}{\sqrt{n+1} - \sqrt{n}} = |x-5| < 1 \Rightarrow R = 1$

$x=6 \Rightarrow \sum_{n=1}^{\infty} (\sqrt{n+1} - \sqrt{n}) = -\sum_{n=1}^{\infty} (\sqrt{n} - \sqrt{n+1})$ diverg telescoping

$x=4 \Rightarrow \sum_{n=1}^{\infty} (\sqrt{n+1} - \sqrt{n})(-1)^n \Rightarrow alt + lim = 0 + dec \Rightarrow$ convg

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1) The sequence $\left\{ \frac{10}{4} - \sqrt{2}, \frac{19}{8} - \sqrt{2\sqrt{2}}, \frac{28}{12} - \sqrt{2\sqrt{2\sqrt{2}}}, \frac{37}{16} - \sqrt{2\sqrt{2\sqrt{2\sqrt{2}}}}, \dots \right\}$

(a) Converges to $7/4$

(b) Converges to $5/4$

(c) Converges to $3/4$

(d) Converges to $1/4$

(e) is divergent

(f) None of the above

$$\left\{ \frac{10}{4}, \frac{19}{8}, \frac{28}{12}, \frac{37}{16}, \dots \right\} = \left\{ \frac{9n+1}{4n} \right\}_{n=1}^{\infty}$$

$$\lim_{n \rightarrow \infty} \frac{9n+1}{4n} = \frac{9}{4}$$

$$\left\{ \sqrt{2}, \sqrt{2\sqrt{2}}, \sqrt{2\sqrt{2\sqrt{2}}}, \dots \right\}$$

$$a_{n+1} = \sqrt{2a_n} \quad n=1, 2, 3, \dots$$

$$a_1 = \sqrt{2}$$

$$\lim_{n \rightarrow \infty} a_{n+1} = \lim_{n \rightarrow \infty} \sqrt{2a_n}$$

$$\lim_{n \rightarrow \infty} a_{n+1} = \sqrt{2 \lim_{n \rightarrow \infty} a_n}$$

$$L = \sqrt{2L}$$

$$\Rightarrow L^2 = 2L$$

$$\Rightarrow L(L-2) = 0$$

$$\Rightarrow L = 2$$

Hence, the limit of the sequence is

$$\frac{9}{4} - 2 = \frac{9}{4} - \frac{8}{4} = \frac{1}{4}$$