Name:	ID:	Sec: 8(9:00AM) 9(10:00AM)

MATH-102 Term-152 Long Class-QUIZ-5 FormB

(circle one letter only)

1) Using four rectangles and the midpoint rule, the area under the graph of $f(x)=1+x^2$ from $x=0$ to $x=4$ is approximately equal to

0 1 2 3 4

$1/2$ $3/2$ $5/2$ $7/2$

$f(x_i)$ $5/4$ $13/4$ $29/4$ $53/4$

(a) 27

(b) 20

(c) 30

(d) 18

(e) 25

(f) None of the above

$M_4 = \Delta x\,[f(x_1)+\dots+f(x_4)]$

$M_4 = (1)[100/4]$

$= 25$

2) $\int_0^1 \frac{e^{\tan^{-1}x}}{1+x^2}dx = I$

$u=\tan^{-1}x \Rightarrow du=\frac{dx}{1+x^2}$

$I=\int_0^{\pi/4} e^u du = [e^u]_0^{\pi/4}$

$= e^{\pi/4}-1$

(a) $1-e^{\pi/2}$

(b) $e^{\pi}-\pi$

(c) $e^{\pi/2}-1$

(d) $e^{\pi/4}-1$

(e) $e^{-\pi/4}-2$

(f) None of the above

3) $\int_0^1 (1+x)\sqrt{4-4x}\,dx = 2\int_0^1 (1+x)\sqrt{1-x}\,dx$

let $u=1-x \Rightarrow du=-dx$

$x=1-u$

$I=-2\int_1^0 (2-u)u^{1/2}du$

$=2\int_0^1 (2u^{1/2}-u^{3/2})du$

$=2\left[\frac{4}{3}u^{3/2}-\frac{2}{5}u^{5/2}\right]_0^1$

$=2\left[\frac{4}{3}-\frac{2}{5}\right]=2\left[\frac{20-6}{15}\right]$

$=28/15$

(a) 14/15

(b) 15/28

(c) 28/15

(d) 28/13

(e) 15/13

(f) None of the above

4) If g is a continuous function such that $\int_0^{2x} e^{t/2}g(t)dt = \frac{1}{2}xe^x$, then $g(4)=$

diff both sides, use FTC

$e^x g(2x)(2) = \frac{1}{2}[e^x + xe^x]$

$g(2x) = \frac{1}{4}[1+x]$

let $x=2$

$g(4)=\frac{1}{4}[3]=3/4$

(a) 4/3

(b) 3/4

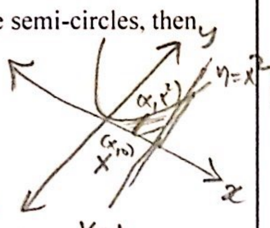
(c) 3/2

(d) 4/5

(e) 2/3

(f) None of the above

5) The base of a solid is bounded by the curves $y=x^2$, $y=0$ and $x=1$. If the cross-sections perpendicular to the x-axis are semi-circles, then the volume of the solid is



$s=x^2$

$r=\frac{1}{2}x^2$

semi-circle

$A=\frac{1}{2}\pi\left(\frac{1}{2}x^2\right)^2$

$=\frac{\pi}{8}x^4$

(a) $4/\pi$

(b) $40/\pi$

(c) $\pi/4$

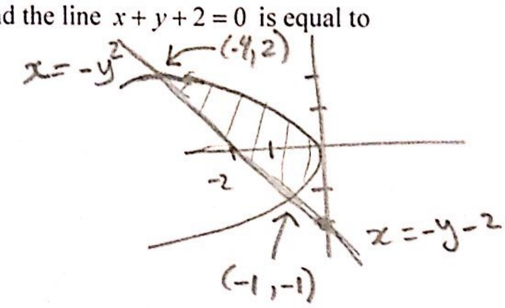
(d) $\pi/40$

(e) 2π

(f) None of the above

$V=\frac{\pi}{8}\int_0^1 x^4 dx = \frac{\pi}{8}\left(\frac{1}{5}\right)=\frac{\pi}{40}$

6) The area of the region enclosed by the curves $y^2=-x$ and the line $x+y+2=0$ is equal to



(a) 9/2

(b) 7/2

(c) 1

(d) 0

(e) 5/2

(f) None of the above

Right to Left

write all curves $x=\cdots$

$A=\int_{-1}^{2}\left[(-y^2)-(-y-2)\right]dy$

$=\left[-\frac{1}{3}y^3+\frac{1}{2}y^2+2y\right]_{-1}^{2} = \left(-\frac{8}{3}+2+4\right)-\left(\frac{1}{3}+\frac{1}{2}-2\right)$

$=8-\frac{8}{3}-\frac{1}{3}-\frac{1}{2}=5-\frac{1}{2}=9/2$

(circle one letter)

7) $\int_{-2}^{2}\left(\frac{x^2\tan x}{x^2+1}+\sqrt{4-x^2}\right)dx =$

(a) 0

(b) $\pi/4$

(c) $\pi/3$

(d) π

(e) $\pi/2$

(f) None of the above

$f(x)=\frac{x^2\tan x}{x^2+1}$ is odd

$f(-x)=\frac{x^2\tan(-x)}{x^2+1}=\frac{-x^2\tan x}{x^2+1}$

$=-f(x) \Rightarrow \int_{-2}^{2}f(x)\,dx=0$

$\int_{-2}^{2}\sqrt{4-x^2}\,dx$ half circle with $r=2$

$\int_{-2}^{2}\sqrt{4-x^2}\,dx=2\pi$

8) The region in the first quadrant enclosed by the parabolas $y=x^2$, $y=2-x^2$, and the y-axis is rotating about the line $y=-1$, then the volume of the solid generated is given by

(a) $\int_0^1 \pi(1-x^4)dx$

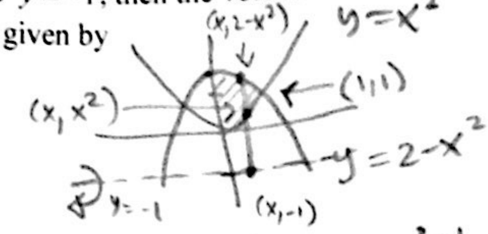
(b) $\int_0^2 8\pi(1+x^2)dx$

(c) $\int_0^1 4\pi(1-x^2)dx$

(d) $\int_0^1 8\pi(1-x^2)dx$

(e) $\int_0^1 4\pi(1+x^2)dx$

(f) None of the above



$r_{out}=3-x^2$, $r_{in}=x^2+1$

$A=\pi\left[(3-x^2)^2-(x^2+1)^2\right]$

$=\pi\left[x^4-6x^2+9-x^4-2x^2-1\right]$

$=\pi\left[8-8x^2\right]=8\pi\left[1-x^2\right]$

9) If f is an even function such that $\int_{-3}^{3}f(t)dt=10$ and $\int_{-2}^{2}f(t)dt=16$, then $\int_{\sqrt{2}}^{\sqrt{3}}2xf(x^2)dx =$ I

(a) 0

(b) 3

(c) -3

(d) 8

(e) -5

(f) None of the above

f even $\Rightarrow \int_0^2=8,\ \int_0^3=5$

$u=x^2 \Rightarrow du=2x\,dx$

$I=\int_2^3 f(u)\,du=5-8=-3$

10) If $F(x)=\int_{1/2}^{2x}f(t)dt$ and $f(t)=\int_{1/2}^{t^2}\frac{\sqrt{1+u^2}}{u}dt$ then $F''(1)=$

(a) $5\sqrt{17}$

(b) $4\sqrt{17}$

(c) $3\sqrt{17}$

(d) $2\sqrt{17}$

(e) $\sqrt{17}$

(f) None of the above

$F'(x)=f(2x)(2)$

$F''(x)=f'(2x)(2)(2)$

$=4f'(2x)$

$f'(t)=\frac{\sqrt{1+t^4}}{t^2}(2t)=\frac{2}{t}\sqrt{1+t^4}$

$f'(2x)=\frac{2}{2x}\sqrt{1+16x^4}=\frac{1}{x}\sqrt{1+16x^4}$

$F''(1)=4f'(2)=(4)\sqrt{17}$

$=4\sqrt{17}$

11) $\int_0^{\frac{\pi}{4}}(\sec x+\cos x)^2dx =$ I

(a) $(5\pi+10)/8$

(b) $(4\pi+10)/8$

(c) $(3\pi+10)/8$

(d) $(5\pi+10)/8$

(e) $(2\pi+10)/8$

(f) None of the above

$(\sec x+\cos x)^2=$

$\sec^2 x+2+\cos^2 x$

$\sec^2 x+2+\frac{1}{2}+\frac{1}{2}\cos 2x$

$I=\left[\tan x+\frac{5}{2}x+\frac{1}{4}\sin 2x\right]_0^{\pi/4}$

$=\left(1+\frac{5\pi}{8}+\frac{1}{4}\right)-(0+0+0)$

$=\frac{5}{4}+\frac{5\pi}{8}=\frac{10}{8}+\frac{5\pi}{8}$

$=(10+5\pi)/8$

12) $\int\frac{\ln(\tan^{-1}x)}{(x^2+1)\tan^{-1}x}dx =$ I

(a) $\frac{1}{2}\left[\ln(\tan^{-1}x)\right]+C$

(b) $\frac{1}{2}\left[\ln(\tan^{-1}x)\right]^2+C$

(c) $\left[\ln(\tan^{-1}x)\right]^2+C$

(d) $\frac{1}{2}\left[\tan^{-1}x\right]^2+C$

(e) $\ln(\tan^{-1}x)+C$

(f) None of the above

$u=\tan^{-1}x \Rightarrow du=\frac{dx}{1+x^2}$

$I=\int\frac{\ln u}{u}du$

$w=\ln u \Rightarrow dw=\frac{du}{u}$

$I=\int w\,dw=\frac{1}{2}w^2+C$

$=\frac{1}{2}(\ln u)^2+C$

$=\frac{1}{2}\left[\ln(\tan^{-1}x)\right]^2+C$