Name:	**ID:**	**Sec**: **34**(11:00-11:50) **23**(1:10-2:00)

MATH-102 **Term-132** **Long Class-QUIZ-5 FormA**

(circle one letter only)

1)The average value of the function $f(x) = \frac{\tan x}{x^2+1} + \sqrt{4-x^2}$ over the interval [-2,2] is

$f_{avg} = \frac{1}{b-a}\int_a^b f(x)\,dx$

(a) 0

(b) π/4

(c) π/3

(d) $\pi/2$

(e) π

(f) None of the above

$= \frac{1}{4}\int_{-2}^{2}\left(\frac{\tan x}{x^2+1} + \sqrt{4-x^2}\right)dx$

$= \frac{1}{4}\left[\int_{-2}^{2}\text{odd} + \int_{-2}^{2}\sqrt{4-x^2}\,dx\right]$

$= \frac{1}{4}\left[0 + \text{half circle}\right]$

$= \frac{1}{4}\left[\frac{1}{2}(4\pi)\right]$

$= \pi/2$

2) $\int_0^1 \frac{e^{\tan^{-1}x}}{1+x^2}dx =$

$u = \tan^{-1}x \to du = \frac{dx}{1+x^2}$

$= \int_0^{\pi/4} e^u\,du = \left[e^u\right]_0^{\pi/4}$

$= e^{\pi/4} - 1$

(a) $1-e^{\pi/2}$

(b) $e^{\pi}-\pi$

(c) $e^{\pi/2}-1$

(d) $e^{-\pi/4}-2$

(e) $e^{\pi/4}-1$

(f) None of the above

3) $\int_0^1 (1+x)\sqrt{4-4x}\,dx =$ $\int_0^1 (1+x)\sqrt{4}\sqrt{1-x}\,dx$

$= 2\int_0^1 (1+x)\sqrt{1-x}\,dx$

Let $u = 1-x \to du = -dx$

$= -2\int_1^0 (2-u)\sqrt{u}\,du$

$= -2\int_1^0 (2u^{1/2} - u^{3/2})\,du$

$= -2\left[\frac{4}{3}u^{3/2} - \frac{2}{5}u^{5/2}\right]_1^0$

$= -2\left[(0) - \left(\frac{4}{3} - \frac{2}{5}\right)\right]$

$= 2\left(\frac{14}{15}\right) = 28/15$

(a) 28/15

(b) 15/28

(c) 14/15

(d) 28/13

(e) 15/13

(f) None of the above

4) If g is a continuous function such that $\int_0^{2x} e^{t/2}g(t)\,dt = \frac{1}{2}xe^x$, then g(4) =

diff w.r.t x both sides

$2e^x g(2x) = \frac{1}{2}(e^x + xe^x)$

$\Rightarrow g(2x) = \frac{1}{4}(1+x)$

let $x = 2$

$g(4) = 3/4$

(a) 4/3

(b) 2/3

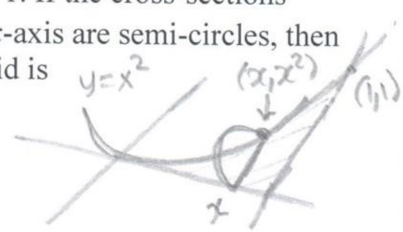
(c) 3/2

(d) 4/5

(e) 3/4

(f) None of the above

5) The base of a solid is bounded by the curves $y = x^2$, $y = 0$ and $x = 1$. If the cross-sections perpendicular to the x-axis are semi-circles, then the volume of the solid is



(a) π/4

(b) 40/π

(c) π/40

(d) 4/π

(e) 2π

(f) None of the above

diameter $= x^2$

radius $= \frac{1}{2}x^2$

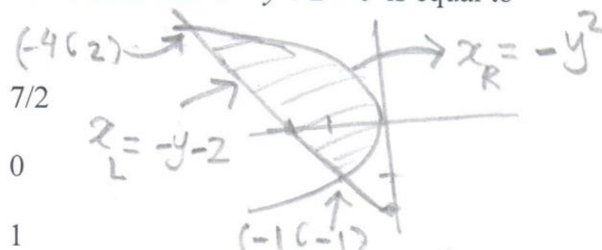
Area of cross section is

$\frac{1}{2}\pi\left(\frac{1}{2}x^2\right)^2 = \frac{\pi}{8}x^4$

$V = \int_0^1 \frac{\pi}{8}x^4\,dx = \frac{\pi}{8}\left[\frac{1}{5}x^5\right]_0^1$

$= \pi/40$

6) The area of the region enclosed by the curves $y^2 = -x$ and the line $x + y + 2 = 0$ is equal to



(a) 7/2

(b) 0

(c) 1

(d) 9/2

(e) 5/2

(f) None of the above

intersection: $y^2 = y+2$

$\to y^2 - y - 2 = 0 \to$

$(y-2)(y+1) = 0$

$A = \int_{-1}^{2}(x_R - x_L)\,dy = \int_{-1}^{2}(-y^2+y+2)\,dy$

$= \left[-\frac{1}{3}y^3 + \frac{1}{2}y^2 + 2y\right]_{-1}^{2} = \left(-\frac{8}{3}+6\right) - \left(\frac{1}{3}+\frac{1}{2}-2\right)$

$= -3 + 8 - \frac{1}{2} = 9/2$

(circle one letter)

7) Let P be a partition of the interval [0, 2], then the limit $\lim_{\|P\|\to 0}\sum_{k=1}^{n}\left[c_k + c_k^2\right]\Delta x_k =$ $R_n = \sum_{k=1}^{n}\Delta x_k f(x_k)$

$= \int_0^2 (x+x^2)\,dx$

$= \left[\frac{1}{2}x^2 + \frac{1}{3}x^3\right]_0^2$

$= (2 + \frac{8}{3}) - (0)$

$= 14/3$

(a) 12/3

(b) 14/3

(c) 16/3

(d) 18/3

(e) 20/3

(f) None of the above

8) The region in the first quadrant enclosed by the parabolas $y = x^2$, $y = 2 - x^2$, and the y-axis is rotating about the line $x = -1$, then the volume of the solid generated is given by

(a) $\int_0^1 \pi(1-x^4)dx$

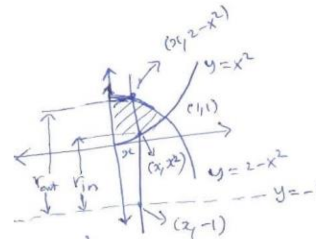
(b) $\int_0^2 8\pi(1+x^2)dx$

(c) $\int_0^1 4\pi(1-x^2)dx$

(d) $\int_0^1 8\pi(1-x^2)dx$

(e) $\int_0^1 4\pi(1+x^2)dx$

(f) None of the above



$r_{out} = 2-x^2+1 = 3-x^2$

$r_{in} = x^2+1$

$A = \pi\left((3-x^2)^2 - (x^2+1)^2\right)$

$= \pi(9-6x^2+x^4-x^4-2x^2-1)$

$= \pi(8-8x^2)$

$V = \int_0^1 \pi(8-8x^2)dx$

9) If f is an even function such that $\int_{-3}^{3} f(t)dt = 10$ and $\int_{-2}^{2} f(t)dt = 16$, then $\int_{-3}^{-2} f(t)dt =$

(a) -3

(b)

(c)

(d)

(e)

(f) None of the above

$\int_{-3}^{3} = 10 \Rightarrow \int_{-3}^{0} = 5$

$\int_{-2}^{2} = 16 \Rightarrow \int_{-2}^{0} = 8$

$\int_{-3}^{0} = \int_{-3}^{-2} + \int_{-2}^{0}$

$5 = \int_{-3}^{-2} + 8$

$\Rightarrow \int_{-3}^{-2} = -3$

10) If $F(x) = \int_{1/2}^{2x} f(t)dt$ and $f(t) = \int_{1/2}^{t^2} \frac{\sqrt{1+u^2}}{u}dt$ then $F''(1) =$

(a) $5\sqrt{17}$

(b) $4\sqrt{17}$

(c) $3\sqrt{17}$

(d) $2\sqrt{17}$

(e) $\sqrt{17}$

(f) None of the above

$F'(x) = 2f(2x)$

Now, $f'(t) = \frac{\sqrt{1+t^4}}{t^2}(2t)$

$F''(x) = (2)f'(2x)(2)$

$F''(x) = 4f'(2x)$

$\Rightarrow F''(1) = 4f'(2) =$

$= 4\left(\frac{\sqrt{17}}{4}\right)(4) = 4\sqrt{17}$

11) $\int_0^{\frac{\pi}{4}} (\sec x + \cos x)^2 dx =$ $\int_0^{\pi/4}(\sec^2 x + 2 + \cos^2 x)$

$= \left[\tan x + 2x\right]_0^{\pi/4} + \int_0^{\pi/4}\cos^2 x\,dx$

$= \left[1 + \frac{\pi}{2}\right] + \int_0^{\pi/4}\left(\frac{1}{2} + \frac{1}{2}\cos 2x\right)$

$= \left[1 + \frac{\pi}{2}\right] + \left[\frac{1}{2}x + \frac{1}{4}\sin 2x\right]_0^{\pi/4}$

$= 1 + \frac{\pi}{2} + \frac{\pi}{8} + \frac{1}{4}$

$= \frac{10}{8} + \frac{5\pi}{8}$

(a) (5π+10)/8

(b) (4π+10)/8

(c) (3π+10)/8

(d) (2π+10)/8

(e) (π+10)/8

(f) None of the above

12) $I = \int \frac{\ln(\tan^{-1}x)}{(x^2+1)\tan^{-1}x}dx =$

(a) $\left[\ln(\tan^{-1}x)\right]^2 + C$

(b) $\frac{1}{2}\left[\ln(\tan^{-1}x)\right] + C$

(c) $\frac{1}{2}\left[\ln(\tan^{-1}x)\right]^2 + C$

(d) $\frac{1}{2}\left[\tan^{-1}x\right]^2 + C$

(e) $\ln(\tan^{-1}x) + C$

(f) None of the above

let $u = \ln(\tan^{-1}x)$

$du = \frac{dx}{(1+x^2)\tan^{-1}x}$

$I = \int u\,du$

$= \frac{1}{2}u^2 + C$

$= \frac{1}{2}\left[\ln(\tan^{-1}x)\right]^2 + C$